Chapter 4

INTEGRATION

GROUP (A)-HOME WORK PROBLEMS

1) $\int\left(x^4+3x^2-2x-5\cos x+7e^x\right)$

Ans. Let I = $\int\left(x^4+3x^2-2x-5\cos x+7e^x\right)$

$$=\frac{x^5}{5}+\frac{3x^3}{3}-\frac{2x^2}{2}-5(\sin x)+7e^x+c$$

$$=\frac{x^5}{5}+x^3-x^2-5\sin x+7e^x+c$$

2) $\int\left(x^7-5x^4+\frac{4}{x^2}-\frac{3}{\sqrt{x}}\right)dx$

Ans. Let, I = $\int\left(x^7-5x^4+\frac{4}{x^2}-\frac{3}{\sqrt{x}}\right)dx$

$$=\frac{x^8}{8}-\frac{5x^5}{5}+4\left(-\frac{1}{x}\right)-3\times2\sqrt{x}+c$$

$$=\frac{x^8}{8}-x^5-\frac{4}{x}-6\sqrt{x}+c$$

3) $\int\begin{bmatrix}4(5+4x)^2-2\sin(3-2x)\\+5\operatorname{cosec}^2\left(3-\frac{x}{5}\right)\end{bmatrix}dx$

Ans. Let,I = $\int\begin{bmatrix}4(5+4x)^2-2\sin(3-2x)\\+5\operatorname{cosec}^2\left(3-\frac{x}{5}\right)\end{bmatrix}dx$

$$=\frac{4(5+4x)^3}{3}\times\frac{1}{4}-\frac{2(-\cos(3-2x))}{-2}+\frac{5\left(-\cot\left(3-\frac{x}{5}\right)\right)}{-\frac{1}{5}}+c$$

$$=\frac{(5+4x)^3}{3}-\cos(3-2x)+25\cot\left(3-\frac{x}{5}\right)+c$$

4) $f'(x) = 4x^3 - 2x + 1$

Ans. We know,

$$f(x)=\int f'(x)\,dx$$

$$=\int\left(4x^3-2x+1\right)dx$$

$$=\frac{4x^4}{4}-\frac{2x^2}{2}+x+c$$

∴ $f(x) = x^4 - x^2 + x + c$ … (i)

∴ $f(x) = 2^4 - 2^2 + 2 + c$

∴ $17 = 16 - 84 + 2 + c$ $\quad[\because f(2)=17]$

∴ $f(x) = x^4 - x^2 + x + 3$ [from i]

GROUP (B)-HOME WORK PROBLEMS

1) $\int\frac{6x-8}{2x-1}dx$

Ans. Let, I = $\int\frac{6x-8}{2x-1}dx$

Here,

$$\frac{6x-8}{2x-1}\Rightarrow 2x-1\overset{\displaystyle 3}{\overline{)6x-8}}$$

$$\begin{array}{r}6x-3\\ -\quad+\\ \hline -5\end{array}$$

∴ $\frac{6x-8}{2x-1}=3-\frac{5}{2x-1}$

So, I = $\int\frac{6x-8}{2x-1}dx$

$$=\int\left(3-\frac{5}{2x-1}\right)dx$$

$$=3\int1.dx-\frac{5}{2}\int\frac{2}{2x-1}dx$$

$$=3x-\frac{5}{2}\log|2x-1|+c$$

2) $\frac{3x^2+4x-2}{x-1}$

Ans. Here, $\frac{3x^2+4x-2}{x-1}$

$$\Rightarrow \begin{array}{r} 3x+7 \\ x-1\overline{)3x^2+4x-2} \\ 3x^2-3x \\ -\quad\; + \\ \hline 7x-2 \\ 7x-7 \\ -\quad + \\ \hline +5 \end{array}$$

Let, I = $\int \frac{3x^2+4x-2}{x-1}dx$

$= \int\left(3x+7+\frac{5}{x-1}\right)dx$

$= \frac{3x^2}{2}+7x+5\log|x-1|+c$

3) $\frac{8x^2+10x-2}{2x+1}$

Ans. Here, $\frac{8x^2+10x-2}{2x+1}$

$$\Rightarrow \begin{array}{r} 4x+3 \\ 2x+1\overline{)8x^2+10x-2} \\ 8x^2+4x \\ -\quad\; - \\ \hline 6x-2 \\ 6x+3 \\ -\quad - \\ \hline -5 \end{array}$$

Let, I = $\int \frac{8x^2+10x-2}{2x+1}dx$

$= \int\left(4x+3-\frac{5}{2x+1}\right)dx$

$= 4\int x\,dx+3\int 1\,.dx-\frac{5}{2}\int\frac{2}{2x+1}dx$

$= \frac{4x^2}{2}+3x-\frac{5}{2}\log|2x+1|+c$

4) $\frac{3x^2+6x-2}{2x-3}$

Ans. Here, $\frac{3x^2+6x-2}{2x-3}$

$$\Rightarrow \begin{array}{r} \frac{3x}{2}+\frac{21}{4} \\ 2x-3\overline{)3x^2+6x-2} \\ 3x^2-\frac{9x}{2} \\ -\quad + \\ \hline \frac{21x}{2}-2 \\ \frac{21x}{2}-\frac{63}{4} \\ -\quad + \\ \hline \frac{55}{4} \end{array}$$

Let, I = $\int \frac{3x^2+6x-2}{2x-3}dx$

$= \int\left(\frac{3x}{2}+\frac{21}{4}+\frac{\frac{55}{4}}{2x-3}\right)dx$

$= \frac{3}{2}\int x\,dx+\frac{21}{4}\int 1.dx+\frac{55}{8}\int\frac{2}{2x-3}dx$

$= \frac{3x^2}{4}+\frac{21x}{4}+\frac{55}{8}\log|2x-3|+c$

5) $\frac{3x^3+7x^2+7x+1}{3x+4}$

Ans. Here, $\frac{3x^3+7x^2+7x+1}{3x+4}$

$$\Rightarrow \begin{array}{r} x^2+3x-\frac{5}{3} \\ 3x+4\overline{)3x^3+7x^2+7x+1} \\ 3x^3+4x^2 \qquad\qquad \\ -\quad\; - \qquad\qquad \\ \hline 3x^2+7x+1 \\ 3x^2+12x \quad \\ -\quad\; - \quad \\ \hline -5x+1 \\ -5x-\frac{20}{3} \\ +\quad\; + \\ \hline \frac{23}{3} \end{array}$$

Let, I = $\int \frac{3x^3+7x^2+7x+1}{3x+4}dx$

$$= \int \left(x^2 + 3x - \frac{5}{3} + \frac{\frac{23}{3}}{3x+4} \right) dx$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} - \frac{5}{3}x + \frac{23}{6}\int \frac{3}{3x+4}\,dx$$

$$= \frac{x^3}{3} + \frac{3x^2}{2} - \frac{5}{3}x + \frac{23}{6}\log|3x+4| + c$$

6) $\frac{3x-1}{x+5}$

Ans. Here, $\frac{3x-1}{x+5}$

$$\begin{array}{r} 3 \\ \Rightarrow x+5\overline{)3x-1} \\ \underset{-}{3x}+\underset{-}{15} \\ \hline -16 \end{array}$$

Let, I = $\int \frac{3x-1}{x+5}\,dx$

$$= \int \left(3 - \frac{16}{x+5}\right) dx$$

$$= 3\int 1.dx 16 \int \frac{1.dx}{x+5}$$

$$= 3x - 16\log|x+5| + c$$

GROUP (C)-HOME WORK PROBLEMS

1) I= $\int (\cos 3x + 4 \sin x - 2 \sin^2 x)\, dx$

Ans. $= \int \cos 3x\, dx + 4 \int \sin x\, dx - 2x$

$$\int \frac{1 - \cos 2x}{2}\, dx.$$

$$= \int \cos 3x\, dx + 4\int \sin x dx - \int 1.dx + \int \cos 2x\, dx$$

$$= \frac{\sin 3x}{3} - 4 \cos x - x + \frac{\sin 2x}{2} + c$$

2) $\int \sin 8x \cos 4x\, dx$

Ans. $= \frac{1}{2} \int 2 \sin 8x \cos 4x\, dx$

$$= \frac{1}{2} \int \sin 12x + \sin 4x\, dx$$

$$= -\frac{\cos 12x}{24} - \frac{\cos 4x}{8} + c$$

3) $\int \sin 3x \times \sin 2x\, dx$

Ans. $= \frac{1}{2} \int 2 \sin 3x \sin 2x\, dx$

$$= \frac{1}{2} \int \cos x - \cos 5x$$

$$= \frac{\sin x}{2} - \frac{\sin 5x}{10} + c$$

4) $\int \cos 7x \cos 5x\, dx$

Ans. $= \frac{1}{2} \int 2 \cos 7x \cos 5x\, dx$

$$= \frac{1}{2} \int \cos 12x + \cos 2x\, dx$$

$$= \frac{\sin 12x}{24} + \frac{\sin 2x}{4} + c$$

5) $\int \cos^2 x \times \sin^2 x\, dx$

Ans. $= \frac{1}{4} \int (2 \sin x \cos x)^2\, dx$

$$= \frac{1}{4} \int \sin^2 2x\, dx$$

$$= \frac{1}{4} \int \frac{1 - \cos 4x}{2}$$

$$= \frac{1}{8} \left[x - \frac{\sin 4x}{4} \right] + c$$

6) $\int (\tan x - \cot x)^2\, dx$

Ans. $= \int \left(\tan^2 x + \cot^2 x - 2 \right) dx$

$$= \int \left(\sec^2 x - 1 + \operatorname{cosec}^2 x - 1 - 2 \right) dx$$

$$= \int \sec^2 x\, dx + \int \operatorname{cosec}^2 x\, dx - \int 4\, dx$$

$$= \tan x - \cot x - 4x + c$$

7) $\int (\tan 2x + \cot 2x)^2\, dx$

Ans. $= \int \left(\tan^2 2x + \cot^2 2x + 2 \right) dx$

$$= \int \sec^2 2x - 1 + \operatorname{cosec}^2 2x - 1 + 2$$

$$= \frac{\tan 2x}{2} - \frac{\cot 2x}{2} + c$$

8) $\int \frac{\sin x + \text{cosec } x}{\tan x} dx$

Ans. $= \int \frac{\sin x + \frac{1}{\sin x}}{\frac{\sin x}{\cos x}} dx$

$= \int \frac{(\sin^2 x + 1)}{\sin^2 x} \cos x\, dx$

$= \int \left(1 + \frac{\text{cosec } x}{\sin x}\right) \cos x\, dx$

$= \int \cos\, dx + \int \text{cosec } x \cot x\, dx$

$= \sin x - \text{cosec } x + c$

9) $\int \left(\frac{e^x \cos x - 4^x \tan x}{4^x \cos x}\right) dx$

Ans. $= \int \left(\frac{e}{4}\right)^x dx - \int \sec x \tan x\, dx$

$= \frac{\left(\frac{e}{4}\right)^x}{\log\left(\frac{e}{4}\right)} - \sec x + c$

$= \frac{e^x}{4^x (1 - \log 4)} - \sec x + c$

10) $\int \frac{e^x \sin x + 3^x \cot x}{3^x \sin x} dx$

Ans. $= \int \left(\frac{e}{3}\right)^x dx + \int \cot x\, \text{cosec } x\, dx$

$= \frac{e^x}{3^x (1 - \log 3)} - \text{cosec } x + c$

11) $\int \frac{\tan x}{\sec x - \tan x} dx$

Ans. $= \int \frac{\sin x}{1 - \sin x} dx$

$= \int \frac{(1 + \sin x) \sin x}{1 - \sin^2 x} dx$

$= \int \frac{\sin x + \sin^2 x}{1 - \sin^2 x} dx$

$= \int \frac{\left(\sin x + \sin^2 x\right)}{\cos^2 x} dx$

$= \int \left(\sec x \tan x + \tan^2 x\right) dx$

$= \int \tan x \sec x\, dx + \int \sec^2 x\, dx - \int 1\, dx$

$= \sec x + \tan x - x + c$

12) $\int \left\{\cos^{-1} (\sin x) + \tan^{-1} (\cot x)\right\} dx$

$= \int \cos^{-1} \left[\cos\left(\frac{\pi}{2} - x\right)\right] + \tan^{-1}\left[\tan\left(\frac{\pi}{2} - x\right)\right] dx$

$= \int \left(\frac{\pi}{2} - x + \frac{\pi}{2} - x\right) dx$

$= \int (\pi - 2x)\, dx$

$= \int \pi\, dx - \int 2x\, dx$

$= x - x^2 + c$

13) $\int \left\{\cos^{-1}(\sin x) + \tan^{-1}(\cot x)\right\} dx$

Same as Q-12

GROUP (D)-HOME WORK PROBLEMS

1) $\int \frac{\sin x}{2 + \cos x} dx$

Ans. Let $f(x) = 2 + \cos x$

$f'(x) = -\sin x$

$= -1 \int \frac{-\sin x}{2 + \cos x} dx$

$= -1 \int \frac{f'(x)}{f(x)} dx$

$= -\log |f(x)| + c$

$= -\log |2 + \cos x| + c$

2) $\int \frac{\cos x}{5 + \sin x}$

Ans. Let $f(x) = 5 + \sin x$

$f'(x) = \cos x$

$\therefore \int \frac{\cos x}{5 + \sin x} dx$

$= \log |5 + \sin x| + c$

3) $\int \frac{1}{\sec x + \tan x} dx$

Ans. $= \int \frac{\cos x}{1 + \sin x} dx$

Let $f(x) = 1 + \sin x$

$f'(x) = \cos x$

$\therefore \quad I = \int \frac{\cos x}{1 + \sin x}$

$= \log|1 + \sin x| + c$

4) $\int \frac{\sin x \cos x}{5\sin^2 x + 2\cos^2 x}\,dx$

Ans. Let $f(x) = 5\sin^2 x + 2\cos^2 x$

$f'(x) = 10 \sin x \cos x - 4 \sin x \cos x$

$= 6 \sin x \cos x$

$\therefore \quad \frac{1}{6}\int \frac{6 \sin x \cos x}{5\sin^2 x + 2\cos^2 x}\,dx$

$= \frac{1}{6}\log|5\sin^2 x + 2\cos^2 x| + c$

5) $\int \frac{\sin 2x}{3 + 2\sin^2 x}\,dx$

Ans. Let $f(x) = 3 + 2\sin^2 x$

$f'(x) = 4 \sin x \cos x = 2 \sin 2x$

$\therefore \quad \frac{1}{2}\int \frac{2 \sin 2x}{3 + 2\sin^2 x}\,dx$

$= \frac{1}{2}\log|3 + 2\sin^2 x| + c$

6) $\int \frac{1}{x + x\log x}\,dx$

Ans. $= \int \frac{1}{x(1 + \log x)}$

Let $f(x) = 1 + \log x$

$f'(x) = \frac{1}{x}$

$\therefore \quad I = \int \frac{1}{x(1 + \log x)}\,dx$

$= \log|1 + \log x| + c$

7) $\int \frac{e^x}{2 + 5e^x}\,dx$

Ans. Let $f(x) = 2 + 5e^x$

$f'(x) = 5e^x$

$\therefore \quad \frac{1}{5}\int \frac{5e^x}{2 + 5e^x}\,dx$

$= \frac{1}{5}\log|2 + 5e^x| + c$

8) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$

Ans. Let $f(x) = e^x - e^{-x}$

$f'(x) = e^x + e^{-x}$

$\therefore \quad I = \int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx$

$= \log|e^x - e^{-x}| + c$

9) $\int \frac{1 + \tan^2 x}{3 + 4\tan x}\,dx$

Ans. $= \int \frac{\sec^2 x}{3 + 4\tan x}\,dx$

Let $f(x) = 3 + 4\tan x$

$f'(x) = 4\sec^2 x$

$\therefore \quad I = \frac{1}{4}\int \frac{4\sec^2 x}{3 + 4\tan x}$

$= \frac{1}{4}\log|3 + 4\tan x| + c$

GROUP (E)-HOME WORK PROBLEMS

1) $\int \frac{7 \sin x + 24 \cos x}{3 \cos x + 4 \sin x}\,dx$

Ans. Let $7 \sin x + 24 \cos x$

$= A[3 \cos x + 4 \sin x] + B[-3 \sin x + 4 \cos x]$

$= (4A - 3B) \sin x + (3A + 4B) \cos x$

$\therefore \quad 4A - 3B = 7$ and $3A + 4B = 24$

on solving, we get,

$A = 4, B = 3$

$\therefore \quad 7 \sin x + 24 \cos x = 4(3 \cos x + 4 \sin x) + 3(4 \cos x - 3 \sin x)$

$\therefore \quad I = \int \frac{4(3 \cos x + 4 \sin x)}{(3 \cos x + 4 \sin x)}\,dx + \int \frac{3(4 \cos x - 3 \sin x)}{(3 \cos x + 4 \sin x)}\,dx$

$= \int 4\,dx + 3\int \frac{4 \cos x - 3 \sin x}{3 \cos x + 4 \sin x}\,dx$

Let $f(x) = 4 \sin x + 3 \cos x$

$f'(x) = 4 \cos x - 3 \sin x$

$\therefore \quad I = 4x + 3\log|3 \cos x + 4 \sin x| + c$

2) $\int \frac{8 \sin x + \cos x}{2 \sin x - 3 \cos x}$

Ans. Let $8 \sin x + \cos x$

$= A[2 \sin x - 3 \cos x] + B[2 \cos x + 3 \sin x]$

$= (2A + 3B) \sin x + (-3A + 2B) \cos x$

$\therefore \quad 2A + 3B = 8$

$-3A + 2B = 1$

On solving, we get,

On solving, we get,

A = 1, B = 2

∴ Nr. = (2 sinx – 3 cosx) + 2 (2 cosx + 3 sinx)

∴ $I = \int 1\ dx + 2 \int \frac{2 \cos x + 3 \sin x}{2 \sin x - 3 \cos x} dx$

$= x + 2 \log |2 \sin x - 3 \cos x| + c$

3) $\int \frac{2 \cos x + 3 \sin x}{6 \sin x - 4 \cos x} dx$

Ans. Nr. = A [Dr.] + B $\left[\frac{d}{dx} (Dr.)\right]$

∴ 2 cosx + 3 sinx = A [6 sinx – 4 cosx] + B [6 cosx + 4 sinx]

∴ 6A + 4B = 3

– 4A + 6B = 2

on solving we get,

$A = \frac{5}{26}$ and $B = \frac{6}{13}$

∴ $2 \cos x + 3 \sin x = \frac{5}{26} (6 \sin x - 4 \cos x) + \frac{6}{13} (6 \cos x + 4 \sin x)$

∴ $I = \int \frac{5}{26} dx + \frac{6}{13} \left[\frac{6 \cos x + 4 \sin x}{6 \sin x - 4 \cos x}\right] dx$

$= \frac{5}{26} x + \frac{6}{13} \log |6 \sin x - 4 \cos x| + c$

4) $\int \frac{4 e^x - 25}{2 e^x - 5} dx$

Ans. Let, $I = \int \frac{4 e^x - 10}{2 e^x - 5} - \frac{15}{2 e^x - 5} dx$

$= \int 2\ dx - \int \frac{15}{2 e^x - 5} dx$

$= 2x - \int \frac{15 e^{-x}}{2 - 5 e^x} dx = 2x - 3 \int \frac{5e^{-x}}{2 - 5e^{-x}} dx$

$= 2x - 3 \log |2 - 5e^{-x}| + c$

5) $\int \frac{20 - 12e^x}{3 e^x - 4} dx$

Ans. $= 2 \int \frac{10 - 6 e^x}{3 e^x - 4} dx = -2 \int \frac{(6e^x - 10)}{3e^x - 4} dx$

$= -2 \int \left[\frac{6 e^x - 8}{3 e^x - 4} - \frac{2}{3 e^x - 4}\right] dx$

$= -2 \left[\int 2\ dx - \int \frac{2 e^{-x}}{3 - 4e^{-x}} dx\right]$

Let $3 - 4e^{-x} = f(x)$

∴ $f'(x) = 4e^{-x}$

$= -2 \left[\int 2\ dx - \frac{1}{2} \int \frac{4 e^{-x}}{3 - 4 e^{-x}} dx\right]$

$= -2 \left[2x - \frac{1}{2} \log |3 - 4 e^{-x}|\right] + c$

6) $\int \frac{3e^{2x} + 5}{4e^{2x} - 5} dx$

Ans. Let $3e^{2x} + 5 = A (4e^{2x} - 5) + B \frac{d}{dx} (4e^{2x} - 5)$

$= A(4e^{2x} - 5) + B (8e^{2x})$

$= 4Ae^{2x} - 5A + 8Be^{2x}$

∴ $3e^{2x} + 5 = (4A + 8B)e^{2x} - 5A$

∴ – 5A = 5

∴ A = – 1

and 4A + 8 B =3

∴ – 4 + 8B = 3

8B = 7 B = 7/8

So,

$I = \int \left[\frac{-1(4e^{2x} - 5) + \frac{7}{8}(8e^{2x})}{4e^{2x} - 5}\right] dx$

$= -\int 1 . dx + \frac{7}{8} \int \frac{8e^{2x}}{4e^{2x} - 5} dx$

$= -x + \frac{7}{8} \log |4e^{2x} - 5| + c$

7) $\int \frac{1}{1 + \tan x} dx$

Ans. $I = \int \frac{1}{1 + \tan x} dx$

$= \int \frac{1}{1 + \frac{\sin x}{\cos x}} dx$

$= \int \frac{\cos x}{\cos x + \sin x} dx$

$= \frac{1}{2} \int \frac{2 \cos x}{\cos x + \sin x} dx$

$= \frac{1}{2} \int \frac{((\cos x + \sin x) + (\cos x - \sin x))}{(\cos x + \sin x)} dx$

$$= \frac{1}{2}\left[\int 1.\ dx + \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx\right]$$

$$= \frac{1}{2}\left[x + \log|\sin x + \cos x|\right] + c$$

8) $\int \frac{1}{1 + \cot x} dx$

Ans. $I = \int \frac{1}{1 + \cot x} dx$

$$= \int \frac{1}{1 + \frac{\cos x}{\sin x}} dx$$

$$= \int \frac{\sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{((\sin x + \cos x) + (\sin x - \cos x))}{(\sin x + \cos x)} dx$$

$$= \frac{1}{2}\left[\int 1. dx - \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx\right]$$

$$= \frac{1}{2}\left[x - \log|\sin x + \cos x|\right] + c$$

9) $\int \frac{2e^x + 3}{2e^x - 3} dx$

Ans. $I = \int \frac{2e^x + 3}{2e^x - 3} dx$

$$= \int \frac{\left(2e^x - 3\right) + (6)}{\left(2e^x - 3\right)} dx$$

$$= \int 1. dx + \int \frac{6}{2e^x - 3} dx$$

$$= x + \int \frac{6e^{-x}}{2 - 3e^{-x}}\ dx$$

$$= x + 2\int \frac{3e^{-x}}{2 - 3e^{-x}}\ dx$$

$$= x + 2 \log\left|2 - 3e^x\right| + c$$

10) $\int \frac{\left(4e^x - 5\right)}{4e^x + 5} dx$

Ans. $I = \int \frac{\left(4e^x - 5\right)}{4e^x + 5} dx$

$$= \int \frac{\left(\left(4e^x + 5\right) - (10)\right)}{\left(4e^x + 5\right)} dx$$

$$= \int 1. dx - \int \frac{10}{4e^x + 5} dx$$

$$= x - \int \frac{10e^{-x}}{1 + 5e^{-x}} dx$$

$$= x + 2 \int \frac{-5e^{-x}}{1 + 5e^{-x}} dx$$

$$= x + 2 \log \left|1 + 5e^x\right| + c$$

GROUP (F)-HOME WORK PROBLEMS

1) $\int \frac{\cos 4x}{\sin 2x} dx$

Ans. $I = \int \frac{\cos 4x}{\sin 2x} dx$

$$= \int \frac{1 - 2\sin^2 2x}{\sin 2x} dx$$

$$= \int \cos ec2x\ dx - 2\int \sin 2x\, dx$$

$$= \log \left|\tan\left(\frac{2x}{2}\right)\right| . \frac{1}{2} - \frac{2(-\cos 2x)}{2} + c$$

$$= \frac{1}{2} \log|\tan x| + \cos 2x + c$$

2) $\int \frac{\sin x}{\sin 2x} dx$

Ans. $I = \int \frac{\sin x}{\sin 2x} dx$

$$= \int \frac{\sin x}{2 \sin x \cos x}\ dx$$

$$= \frac{1}{2} \int \sin x\ dx$$

$$= \frac{1}{2} \log|\sec x + \tan x| + c$$

3) $\int \frac{\cos(x + a)}{\cos x} dx$

Ans. $= \int \frac{\cos x \cos a - \sin x \sin a}{\cos x} dx$

$= \cos a \int 1 \, dx - \sin a \int \tan x \, dx$

$= x \cos a - \sin a \cdot \log |\sec x| + c$

4) $\int \frac{\sin(x + a)}{\cos x} dx$

Ans. $= \int \frac{\sin x \cos a + \cos x \sin a}{\sin x} dx$

$= \cos a \int 1 \, dx + \sin a \int \cot x \, dx$

$= x \cos a + \sin a \cdot \log |\sin x| + c$

5) $\int \frac{\cos(x + a)}{\sin x} dx$

Ans. $= \int \frac{\cos x \cos a - \sin x \sin a}{\sin x} dx$

$= \cos a \int \cot x \, dx - \sin a \int 1 \, dx$

$= \cos a \log |\sin x| - x \sin a + c$

6) $\int \frac{\sin(x + a) \, dx}{\sin(x - a)} dx$

Ans. $= \int \frac{\sin(x - a + 2a)}{\sin(x - a)} dx$

$= \int \frac{\sin(x - a) \cos 2a + \sin 2a \cos(x - a)}{\sin(x - a)} dx$

$= \cos 2a \int 1 \, dx + \sin 2a \int \cot(x - a) \, dx$

$= x \cos 2a + \sin 2a \log |\sin(x - a)| + c$

7) $\int \frac{\cos(x + a)}{\cos(x - b)} dx$

Ans. $= \int \frac{\cos((x-b)+(a+b))}{\cos(x-b)} dx$

$= \int \frac{\cos(x - b) \cos(a + b) - \sin(x - b) \sin(a + b)}{\cos(x - b)} dx$

$= \cos(a + b) \int 1 \, dx - \sin(a + b) \int \tan(x - b) \, dx$

$= x \cos(a + b) - \sin(a + b) \log |\sec(x - b)| + c$

8) $\int \frac{\cos(x + 2a)}{\cos(x - 2a)} dx$

Ans. $= \int \frac{\cos(x - 2a + 4a)}{\cos(x - 2a)} dx$

$=$

$\int \frac{\cos(x - 2a) \cos 4a - \sin(x - 2a) \sin 4a}{\cos(x - 2a)} dx$

$= \cos 4a \int 1 \, dx - \sin 4a \int \tan(x - 2a) \, dx$

$= x \cos 4a - \sin 4a \log |\sec(x - 2a)| + c$

9) $\int \frac{\sin(x + 2a)}{\sin(x + a)} dx$

Ans. $= \int \frac{\sin(x + a + a)}{\sin(x + a)} dx$

$= \int \frac{\sin(x + a) \cos a + \cos(x + a) \sin a}{\sin(x + a)} dx$

$= \cos a \int 1 \, dx + \sin a \int \cot(x + a) \, dx$

$= x \cos a + \sin a \log |\sin(x + a)| + c$

10) $\int \frac{\cos x}{\cos(x + 2a)} dx$

Ans. $= \int \frac{\cos(x + 2a - 2a)}{\cos(x + 2a)} dx$

$= \int \frac{\cos(x + 2a) \cos 2a + \sin(x + 2a) \sin 2a}{\cos(x + 2a)} dx$

$= \cos 2a \int 1 \, dx + \sin 2a \int \tan(x + 2a) \, dx$

$= x \cos 2a + \sin 2a \log |\sec(x + 2a)| + c$

11) $\int \frac{1}{\sin x + \cos x} dx$

Ans. Let, $I = \int \frac{1}{\sin x + \cos x} dx$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x} dx$$

$$= \sqrt{2} \int \frac{1}{\cos \frac{\pi}{4} \sin x + \sin \frac{\pi}{4} \cos x} dx$$

$$= \sqrt{2} \int \frac{1}{\sin\left(x + \frac{\pi}{4}\right)} dx$$

$$= \sqrt{2}\int \cos ec\left(x + \frac{\pi}{4}\right)dx$$

$$= \sqrt{2}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)\right| + c$$

12) $\int \frac{1}{\sin x + \sqrt{3}\cos x}dx$

Ans. Let, I = $\int \frac{1}{\sin x + \sqrt{3}\cos x}dx$

$$= \frac{1}{2}\int \frac{1}{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x}dx$$

$$= \frac{1}{2}\int \frac{1}{\cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x}dx$$

$$= \frac{1}{2}\int \frac{1}{\sin\left(x + \frac{\pi}{3}\right)}dx$$

$$= \frac{1}{2}\int \cos ec\left(x + \frac{\pi}{3}\right)dx$$

$$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2} + \frac{\pi}{6}\right)\right| + c$$

13) $\int \frac{1}{3\sin x - 4\cos x}dx$

Ans. Let, I $= \int \frac{1}{3\sin x - 4\cos x}dx$

$$= \frac{1}{5}\int \frac{dx}{\frac{3}{5}\sin x \frac{-4}{5}\cos x}$$

Let, $\frac{3}{5}$ = cos a and $\frac{4}{5}$ = sin a

∴ tan a = $\frac{\sin\alpha}{\cos\alpha} = \frac{4}{5}$

∴ a = $\tan^{-1}(4/3)$

So,

$$I = \frac{1}{5}\int \frac{1}{\cos\alpha\sin x - \sin\alpha\cos x}dx$$

$$= \frac{1}{5}\int \frac{1}{\sin(x-\alpha)}dx$$

$$= \frac{1}{5}\int \cos ec(x-\alpha)dx$$

$$= \frac{1}{5}\log\left|\tan\left(\frac{x-\alpha}{2}\right)\right| + c$$

$$= \frac{1}{5}\log\left|\tan\left(\frac{x - \tan^{-1}(4/3)}{2}\right)\right| + c$$

14) $\int \frac{1}{\cos(x-a)\sin(x-b)}dx$

Ans. Let, I = $\int \frac{1}{\cos(x-a)\sin(x-b)}dx$

$$= \frac{1}{\cos(b-a)}\int \frac{\cos(b-a)}{\cos(x-a)\sin(x-b)}dx$$

$$= \frac{1}{\cos(b-a)}\int \frac{\cos\left[(x-a)-(x-b)\right]}{\cos(x-a)\sin(x-b)}dx$$

$$= \frac{1}{\cos(b-a)}$$

$$\int \frac{(\cos(x-a)\cos(x-b) + \sin(x-a)\sin(x-b))}{\cos(x-a)\sin(x-b)}dx$$

$$= \frac{1}{\cos(b-a)}\left[\int \cot(x-b)dx + \int \tan(x-a)dx\right]$$

$$= \frac{1}{\cos(b-a)}\left[\log|\sin(x-b)| + \log|\sec(x-a)|\right] + c$$

$$= \frac{1}{\cos(b-a)}\left[\log|\sin(x-b)| - \log|\cos(x-a)|\right] + c$$

$$= \frac{1}{\cos(b-a)}\log\left|\frac{\sin(x-b)}{\cos(x-a)}\right| + c$$

GROUP (G)-HOME WORK PROBLEMS

1) $\int \frac{1}{9x^2 + 6x + 5}dx$

Ans. $= \frac{1}{9}\int \frac{1}{x^2 + \frac{2}{3}x + \frac{5}{9}}dx$

$$= \frac{1}{9}\int \frac{1dx}{x^2 + \frac{2x}{3} + \frac{1}{9} + \frac{5}{9} - \frac{1}{9}}$$

$$= \frac{1}{9}\int \frac{1}{\left(x + \frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}dx$$

$$= \frac{1}{9} \; \frac{1}{2/3} \cdot \tan^{-1}\left(\frac{x + 1/3}{2/3}\right) + c$$

$$= \frac{1}{6} \; \tan^{-1}\left(\frac{3x + 1}{2}\right) + c$$

2) $\displaystyle\int \frac{1}{x^2 + x + 1}\, dx$

Ans. $\displaystyle = \int \frac{1}{x^2 + x + \frac{1}{4} + 1 - \frac{1}{4}}$

$$= \int \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\, dx$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} \; . \; \tan^{-1}\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + c$$

3) $\displaystyle\int \frac{5}{4 - 2x - x^2}\, dx$

Ans. $\displaystyle = -\,5 \int \frac{1}{x^2 + 2x - 4}\, dx$

$$= -\,5 \int \frac{1}{x^2 + 2x + 1 - 5}\, dx$$

$$= -5 \int \frac{1}{(x + 1)^2 - \left(\sqrt{5}\right)^2}\, dx$$

$$= +\,5 \int \frac{1}{\left(\sqrt{5}\right)^2 - (x + 1)^2}\, dx$$

$$= 5 \times \frac{1}{2\sqrt{5}} \; \log \left|\frac{\sqrt{5} + x + 1}{\sqrt{5} - x - 1}\right| + c$$

$$= \frac{\sqrt{5}}{2} \log\left|\frac{\sqrt{5} + x + 1}{\sqrt{5} - x - 1}\right| + c$$

4) $\displaystyle\int \frac{1}{x^2 - 6x - 7}\, dx$

Ans. $\displaystyle = \int \frac{1}{x^2 - 6x + 9 - 16}\, dx$

$$= \int \frac{1}{(x - 3)^2 - 4^2}\, dx$$

$$= \frac{1}{2(4)} \log \left|\frac{(x - 3) - 4}{(x - 3) + 4}\right| + c$$

$$= \frac{1}{8} \; \log \left|\frac{x - 7}{x + 1}\right| + c$$

5) $\displaystyle\int \frac{1}{3x^2 + x + 2}\, dx$

Ans. $\displaystyle = \frac{1}{3} \int \frac{1}{x^2 + \frac{x}{3} + \frac{2}{3}}\, dx$

$$= \frac{1}{3} \int \frac{1}{x^2 + \frac{x}{3} + \frac{1}{36} - \frac{1}{36} + \frac{2}{3}}\, dx$$

$$= \frac{1}{3} \int \frac{1}{\left(x + \frac{1}{6}\right)^2 + \frac{23}{36}}\, dx$$

$$= \frac{1}{3} \int \frac{1}{\left(x + \frac{1}{6}\right)^2 + \left(\frac{\sqrt{23}}{6}\right)^2}\, dx$$

$$= \frac{1}{3} \times \frac{1}{\frac{\sqrt{23}}{6}} \; \tan^{-1} \frac{\left(x + \frac{1}{6}\right)}{\frac{\sqrt{23}}{6}} + c$$

$$= \frac{2}{\sqrt{23}} \; \tan^{-1}\left(\frac{6x + 1}{\sqrt{23}}\right) + c$$

6) $\displaystyle\int \frac{1}{5x^2 - 6x - 8}\, dx$

Ans. $\displaystyle = \frac{1}{5} \int \frac{1}{x^2 - \frac{6x}{5} - \frac{8}{5} - \frac{9}{25} + \frac{9}{25}}\, dx$

$$= \frac{1}{5} \int \frac{1}{\left(x - \frac{3}{5}\right)^2 - \left(\sqrt{\frac{49}{25}}\right)^2}\, dx$$

$$= \frac{1}{5} \int \frac{1}{\left(x - \frac{3}{5}\right)^2 - \left(\frac{7}{5}\right)^2}\, dx$$

$$= \frac{1}{5} \times \frac{1}{2\times\frac{7}{5}} \log \left|\frac{x-\frac{3}{5}-\frac{7}{5}}{x-\frac{3}{5}+\frac{7}{5}}\right| + c$$

$$= \frac{1}{14} \log\left|\frac{x-2}{x+\frac{4}{5}}\right| + c$$

$$= \frac{1}{14} \log \left|\frac{5x-10}{5x+4}\right| + c$$

GROUP (H)-HOME WORK PROBLEMS

1) $\int (3x^2 + 4x - 3)^{3/4} (3x + 2)\, dx$

Ans. Let $(3x^2 + 4x - 3) = t$

diff. w.r.t x.

$(6x + 4)\, dx = dt$

$\therefore$ $2(3x + 2)\, dx = dt$

$$(3x + 2)\, dx = \frac{dt}{2}$$

$$\therefore \int t^{3/4} \frac{dt}{2}$$

$$= \frac{1}{2} \int t^{3/4} dt$$

$$= \frac{1}{2} \frac{t^{7/4}}{\frac{7}{4}} + c$$

$$= \frac{2\,(3x^2 + 4x - 3)^{7/4}}{7} + c$$

2) $\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$

Ans. $I = \int \frac{dx}{\sqrt{\sin^3 x \sin(x + \alpha)}}$

$$= \int \frac{dx}{\sqrt{\sin^3 x (\sin x \cos\alpha + \cos x \sin\alpha)}}$$

$$= \int \frac{dx}{\sqrt{\sin^4 x \cos\alpha + \sin^3 x \cos x \sin\alpha}}$$

$$= \int \frac{dx}{\sqrt{\sin^4 x (\cos\alpha + \cot x \sin\alpha)}}$$

$$= \int \frac{\operatorname{cosec}^2 x \;\; dx}{\sqrt{\cos\alpha + \cot x \sin\alpha}}$$

put $\cos\alpha + \sin x \cos\alpha = t$

$\therefore$ $-\operatorname{cosec}^2 x \sin\alpha\, dx = dt$

$\therefore$ $\operatorname{cosec}^2 x \;\; dx = -\, dt / \sin x$

So,

$$I = -\frac{1}{\sin\alpha} \int \frac{dt}{\sqrt{t}} = -\frac{1}{\sin\alpha} \times 2\sqrt{t} + c$$

$$= -\frac{2}{\sin\alpha} \sqrt{\cos\alpha + \cos x \sin\alpha} + c$$

3) $\int \frac{\sec^2 x}{9 - 5\tan^2 x} dx$

Ans. Put $\tan x = t$

$\therefore$ $\sec^2 x\, dx = dt$

$$\therefore \quad I = \int \frac{dt}{9 - 5t^2} \;\; dt$$

$$= \frac{1}{5} \int \frac{1}{\frac{9}{5} - t^2} dt$$

$$= \frac{1}{5} \int \frac{1}{\left(\frac{3}{\sqrt{5}}\right)^2 - (t)^2} dt$$

$$= \frac{1}{5} \times \frac{1}{2 \times \frac{3}{\sqrt{5}}} \log \left| \frac{\frac{3}{\sqrt{5}} + t}{\frac{3}{\sqrt{5}} - t} \right| + c$$

$$= \frac{1}{6\sqrt{5}} \log \left| \frac{3 + \sqrt{5}\, t}{3 - \sqrt{5}\, t} \right| + c$$

$$= \frac{1}{6\sqrt{5}} \log \left| \frac{3 + \sqrt{5} \tan x}{3 - \sqrt{5} \tan x} \right| + c$$

4) $\int (1 + \cot^3 x) \operatorname{cosec}^2 x\, dx$

Ans. Let $\cot x = t$

$\therefore$ $-\operatorname{cosec}^2 x\, dx = dt$

$$\int (1 + t^3)\, (-\, dt)$$

$$= -\int 1 . dt - \int t^3\, dt$$

$$= -t - \frac{t^4}{4} + c$$

$$= -\cot x - \frac{\cot^4 x}{4} + c$$

5) $\int \frac{1}{(3\tan x + 1)\cos^2 x} \;\; dx$

Ans. $= \int \frac{\sec^2 x}{3\tan x + 1} \;\; dx$

Let $\tan x = t$

$\therefore$ $\sec^2 x\, dx = dt$

$$\therefore \int \frac{dt}{3t + 1} = \frac{\log|3t + 1|}{3} + c$$

$$= \frac{1}{3} \log |3\tan x + 1| + c$$

6) $\int (1 - \sin x)^3 \cos x \, dx$

Ans. Let sinx = t

$\cos x \, dx = dt$

$\therefore \quad \int (1 - t)^3 \, dt$

$= \dfrac{(1 - t)^4}{4(-1)} + c$

$= \dfrac{-1}{4} (1 - \sin x)^4 + c$

7) $\int \dfrac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$

Ans. Let $e^x + e^{-x} = t$

$(e^x - e^{-x}) \, dx = dt$

$\therefore \quad \dfrac{dt}{t} = \log|t| + c$

$= \log |e^x + e^{-x}| + c$

8) $\dfrac{e^x (1 + x)}{\sin^2 (x e^x)} + dx$

Let $xe^x = t \quad \therefore \quad (x . e^x + e^x) \, dx = dt$

$\therefore \quad e^x(1 + x) \, dx = dt$

$\therefore \quad I = \int \dfrac{dt}{\sin^2 t}$

$\therefore \quad \int \operatorname{cosec}^2 t \, dt$

$= -\cot t + c$

$= -\cot (xe^x) + c$

9) $\int \dfrac{(3 - 2 \log x)^{5/2}}{x} \, dx$

Ans. Let $(3 - 2\log x) = t$

$\therefore \quad -\dfrac{2}{x} \, dx = dt$

$\dfrac{1}{x} \, dx = -\dfrac{dt}{2}$

$\therefore \quad I = \int t^{5/2} \left(-\dfrac{dt}{2}\right)$

$= \dfrac{-1}{2} \int t^{5/2} \, dt$

$= \dfrac{-1}{2} \dfrac{t^{7/2}}{\frac{7}{2}} + c$

$= \dfrac{-(3 - 2 \log x)^{7/2}}{7} + c$

10) $\int \dfrac{(\sin^{-1} x)^2}{\sqrt{1 - x^2}} \, dx$

Ans. Let $\sin^{-1}x = t$

$\therefore \quad \dfrac{1}{\sqrt{1 - x^2}} \, dx = dt$

$\therefore \quad I = \int t^2 \, dt$

$= \dfrac{t^3}{3} + c$

$= \dfrac{(\sin^{-1} x)^3}{3} + c$

11) $\int \dfrac{e^{\tan^{-1} 2x}}{1 + 4x^2} \, dx$

Ans. Let $\tan^{-1}2x = t$

$\therefore \quad \dfrac{1}{1 + 4x^2} (2) \, dx = dt$

$\therefore \quad \dfrac{1}{1 + 4x^2} \, dx = \dfrac{dt}{2}$

$\therefore \quad I = \int e^t \times \dfrac{dt}{2}$

$= \dfrac{e^t}{2} + c$

$= \dfrac{e^{\tan^{-1} 2x}}{2} + c$

GROUP (I)-HOME WORK PROBLEMS

1) $\int \dfrac{4x + 7}{2x^2 - 3} \, dx$

Ans. $= \int \dfrac{4x}{2x^2 - 3} \, dx$

$= \int \dfrac{4x}{2x^2 - 3} \, dx + \int \dfrac{7}{2x^2 - 3} \, dx + c$

$I = \int \dfrac{4x}{2x^2 - 3} \, dx + \dfrac{7}{2} \int \dfrac{1}{(x)^2 - \left(\sqrt{\dfrac{3}{2}}\right)^2}$

$= \log |2x^2 - 3| + \dfrac{1 \times 7}{2.2 \dfrac{\sqrt{3}}{\sqrt{2}}} \log \left| \dfrac{x - \dfrac{\sqrt{3}}{\sqrt{2}}}{x + \dfrac{\sqrt{3}}{\sqrt{2}}} \right| + c$

$$= \log\left|2x^2-3\right| + \frac{7}{2\sqrt{6}}\log\left|\frac{\sqrt{2}x-3}{\sqrt{2}x+3}\right|$$

2) $\int \frac{x-1}{x^2+3}\,dx$

Ans. $= \int \frac{x}{x^2+3}\,dx - \int \frac{1}{x^2+3}\,dx$

$\therefore$ $I = \frac{1}{2}\int \frac{2x}{x^2+3}\,dx - \int \frac{1}{x^2+(\sqrt{3})^2}\,dx$

$= \frac{1}{2}\log|x^2+3| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

3) $\int \frac{3x-4}{1-x^2}\,dx$

Ans. $I = \int \frac{3x}{1-x^2}\,dx - \int \frac{4}{1-x^2}\,dx$

$= \frac{-3}{2}\int \frac{-2x}{1-x^2}\,dx - 4\int \frac{1}{(1)^2-x^2}\,dx$

$= \frac{-3}{2}\log\left|1-x^2\right| - 4\times\frac{1}{2(1)}\log\left|\frac{1+x}{1-x}\right| + c.$

$= \frac{-3}{2}\log\left|1-x^2\right| - 2\log\left|\frac{1+x}{1-x}\right| + c$

4) $\int \frac{x^2+1-2}{x^2+1}\,dx$

Ans. $= \int 1\,dx - 2\int \frac{1}{x^2+1^2}\,dx$

$= x - \frac{2}{(1)}\tan^{-1}\left(\frac{x}{1}\right) + c$

$= x - 2\tan^{-1}(x) + c$

5) $\int \frac{\left(x^3+1\right)}{x^2+1}\,dx$

Ans. Here,

$$\begin{array}{r|l} & \qquad\qquad\qquad x \\ \hline x^2+1 & x^3+0x^2+0x+1 \\ & \underline{\underset{-}{x^3} \qquad\quad \pm x} \\ & \qquad\qquad -x+1 \end{array}$$

So, $\frac{x^3+1}{x^2+1} = x + \frac{(-x+1)}{x^2+1}$

So,

$I = \int\left(x - \frac{x}{x^2+1} + \frac{1}{x^2+1}\right)dx$

$= \int x\,dx - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{1^2+x^2}\,dx$

$= \frac{x^2}{2} - \frac{1}{2}\log\left|x^2+1\right| + \tan^{-1}x + c$

6) $\int \frac{x^3-x+1}{x^2-4}$

Ans. Here,

$$\begin{array}{r|l} & \qquad\qquad\qquad x \\ \hline x^2-4 & x^3+0x^2-x+1 \\ & \underline{\underset{-}{x^3} \qquad\quad \mp 4x} \\ & \qquad\qquad 3x+1 \end{array}$$

So, $\frac{x^3-x+1}{x^2-4} = x + \frac{(3x+1)}{x^2-4}$

So, $I = \int\left(x + \frac{3x+1}{x^2-4}\right)dx$

$= \int x\,dx + \int \frac{3x}{x^2-4}\,dx + \int \frac{1}{x^2-2^2}\,dx$

$= \int x\,dx + \frac{3}{2}\int \frac{2x}{x^2-4}\,dx + \int \frac{1}{x^2-2^2}\,dx$

$= \frac{x^2}{2} + \frac{3}{2}\log\left|x^2-4\right| + \frac{1}{2(2)}\log\left|\frac{x-2}{x+2}\right| + c$

$= \frac{x^2}{2} + \frac{3}{2}\log\left|x^2-4\right| + \frac{1}{4}\log\left|\frac{x-2}{x+2}\right| + c$

GROUP (J)-HOME WORK PROBLEMS

1) $\int \sin^3 x\,dx$

Ans. $= \int \sin^2 x \sin x\,dx$

$= \int\left(1-\cos^2 x\right)\sin x\,dx$

Let cos t

$\therefore$ $-\sin x\,dx = dt$

$\therefore$ $I = -\int(1-t^2)\,dt$

$= -\int 1.dt + \int t^2\,dt$

$= -t + \frac{t^3}{3} + c$

$= -\cos x + \frac{\cos^3 x}{3} + c$

2) $\int \cos^3 x\, dx$

Ans. $= \int \cos^2 x \cos x\, dx$

$= \int \left(1 - \sin^2 x\right) \cos x\, dx$

Let sinx = t

$\therefore$ $\cos x\, dx = dt$

$\therefore$ $I = \int (1 - t^2)\, dt$

$= \int 1.dt - \int t^2\, dt$

$= + t - \frac{t^3}{3} + c$

$= \sin x - \frac{\sin^3 x}{3} + c$

3) $I = \int \sin^4 x\, dx$

Ans. $= \int \left(\sin^2 x\right)^2 dx$

$= \int \left(\frac{1 - \cos 2x}{2}\right)^2 dx$

$= \int \frac{1 - 2\cos 2x + \cos^2 2x}{4} dx$

$= \frac{1}{4} \int 1 - 2\cos 2x + \frac{1 + \cos 4x}{2} dx$

$= \frac{1}{4} \int \frac{2 - 4\cos 2x + 1 + \cos 4x}{2}$

$= \frac{1}{8} \int (3 - 4\cos 2x + \cos 4x)\, dx$

$= \frac{3}{8} x - \frac{4}{8} \frac{\sin 2x}{2} + \frac{1}{32} \sin 4x + c$

$= \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$

4) $I = \int \cot^4 x\, dx$

Ans. $= \int \cot^2 x \cdot \cot^2 x\, dx$

$= \int (\mathrm{cosec}^2 x - 1) \cot^2 x \cdot dx$

$= \int \mathrm{cos\,ec}^2 x \cot^2 x - \int \cot^2 x\, dx$

$= \int (\mathrm{cosec}^2 x \cot^2 x)\, dx - \int \mathrm{cosec}^2 x\, dx + \int 1\, dx$

Let t = cot x

$\therefore$ $-\mathrm{cosec}^2 x\, dx = dt$

$\therefore$ $I = \int -t^2\, dt - \int -dt + \int 1\, dx$

$= \frac{-t^3}{3} + t + x + c$

$= -\frac{\cot^3 x}{3} + \cot x + x + c$

5) **Same as Q-4**

6) $\int \tan^3 x\, dx$

Ans. $= \int \tan^2 x \cdot \tan x\, dx = \int \left(\sec^2 x - 1\right) \tan x\, dx$

$= \int \sec^2 x \tan x\, dx - \int \tan x\, dx$

Let tanx = t

$\sec^2 x\, dx = dt$

$\therefore$ $I = \int t\, dt - \int \tan x\, dx$

$= \frac{t^2}{2} - \log|\sec x| + c$

$= \frac{\tan^2 x}{2} - \log |\sec x| + c$

7) $\int \sec^4 x\, dx$

Ans. $= \int \sec^2 x \cdot \sec^2 x\, dx = \int \sec^2 x \left(1 + \tan^2 x\right) dx$

$= \int \sec^2 x\, dx + \int \sec^2 x \tan^2 x\, dx$

put tan x = t

$\therefore$ $\sec^2 x\, dx = dt$

$\therefore$ $I = \int \sec^2 x\, dx + \int t^2 dt$

$= \tan x + \frac{t^3}{3} + c$

$= \tan x + \frac{\tan^3 x}{3} + c$

8) $\int \mathrm{cosec}^4 x\, dx$

Ans. $= \int \mathrm{cosec}^2 x\, \mathrm{cosec}^2 x\, dx$

$= \int (\cot^2 x + 1)\, \mathrm{cosec}^2 x\, dx$

$= \int \cot^2 x\, \mathrm{cosec}^2 x\, dx + \int \mathrm{cosec}^2 x\, dx$

put $\cot x = t$

$\therefore$ $-\operatorname{cosec}^2 x \, dx = dt$

$\therefore$ $\int -t^2 dt + (-\cot x) + c$

$$= \frac{-t^3}{3} - \cot x + c$$

$$= -\frac{\cot^3 x}{3} - \cot x + c$$

GROUP (K)-HOME WORK PROBLEMS

1) $\int \frac{dx}{\sqrt{5 + 4x - x^2}}$

$$I = \int \frac{dx}{\sqrt{5 + 4x - x^2}}$$

$$= \int \frac{1}{\sqrt{5 - \left(x^2 - 4x + 4\right) + 4}} dx$$

$$= \int \frac{1}{\sqrt{9 - (x-2)^2}} dx$$

$$= \int \frac{1}{\sqrt{(3)^2 - (x-2)^2}} dx$$

$$= \sin^{-1}\left(\frac{x-2}{3}\right) + c$$

2) $\int \frac{1}{\sqrt{x^2 - 4x - 5}} dx$

Ans. $I = \int \frac{1}{\sqrt{x^2 - 4x - 5}} dx$

$$= \int \frac{dx}{\sqrt{x^2 - 4x + 4 - 5 - 4}}$$

$$= \int \frac{1}{\sqrt{(x-2)^2 - (3)^2}} dx$$

$$= \log\left|x - 2 + \sqrt{x^2 - 4x - 5}\right| + c$$

3) $\int \frac{1}{\sqrt{x^2 - 4x + 13}} dx$

Ans. $= \int \frac{1}{\sqrt{(x-2)^2 + 3^2}} dx$

$$= \log\left|x - 2 + \sqrt{x^2 - 4x + 13}\right| + c$$

4) $\int \frac{dx}{\sqrt{2x^2 + 7x - 5}}$

Ans. $I = \int \frac{dx}{\sqrt{2x^2 + 7x - 5}}$

$$= \int \frac{dx}{\sqrt{2\left(x^2 + \frac{7}{2}x - \frac{5}{2}\right)}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x^2 + \frac{7x}{2} + \frac{49}{16} - \frac{5}{2} - \frac{49}{16}\right)}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x + \frac{7}{4}\right)^2 - \frac{89}{16}}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x + \frac{7}{4}\right)^2 - \left(\frac{\sqrt{89}}{4}\right)^2}} dx$$

$$= \frac{1}{\sqrt{2}} \times \log\left|x + \frac{7}{4} + \sqrt{x^2 + \frac{7x}{2} - \frac{5}{2}}\right| + c$$

5) $\int \frac{dx}{\sqrt{5 - 7x - 2x^2}}$

Ans. $I = \int \frac{dx}{\sqrt{5 - 7x - 2x^2}}$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - \frac{7x}{2} - x^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - \left(x^2 + \frac{7}{2}x\right)}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{5}{2} - \left(x^2 + \frac{7}{2}x + \frac{49}{16}\right) + \frac{49}{16}}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{89}{16} - \left(x - \frac{7}{4}\right)^2}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{89}}{4}\right)^2 - \left(x - \frac{7}{4}\right)^2}} + c$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x - \frac{7}{4}}{\frac{\sqrt{89}}{4}}\right) + c$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4x - 7}{\sqrt{89}}\right) + c$$

6) $\int \frac{1}{\sqrt{2x^2 + 7x + 13}}\ dx$

Ans. $= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + \frac{7x}{2} + \frac{13}{2}}}\ dx$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + \frac{7}{2}x + \frac{49}{16} - \frac{49}{16} + \frac{13}{2}}}\ dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x + \frac{7}{4}\right)^2 + \frac{55}{16}}}\ dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x + \frac{7}{4}\right)^2 + \left(\frac{\sqrt{55}}{4}\right)^2}}\ dx$$

$$= \frac{1}{\sqrt{2}} \log \left| x + \frac{7}{4} + \sqrt{x^2 + \frac{7x}{2} + \frac{13}{2}} \right| + c$$

7) $\int \frac{e^x\ dx}{\sqrt{e^{2x} - 1}}$

Ans. Let $e^x = t$

$e^x\,dx = dt$

$\therefore$ $I = \int \frac{dt}{\sqrt{t^2 - 1}}$

$$= \log \left| t + \sqrt{t^2 - 1} \right| + c$$

$$= \log \left| e^x + \sqrt{e^{2x} - 1} \right| + c$$

8) $\int \frac{e^x\ dx}{\sqrt{5 - 4e^x - e^{2x}}}$

Ans. Let $e^x = t$

$\therefore$ $e^x\,dx = dt$

$\therefore$ $I = \int \frac{dt}{\sqrt{5 - 4t - t^2}}$

$$= \int \frac{dt}{\sqrt{5 - \left(t^2 + 4t + 4\right) + 4}}$$

$$= \int \frac{dt}{\sqrt{9 - (t+2)^2}}$$

$$= \int \frac{1}{\sqrt{3^2 - (t+2)^2}} = \sin^{-1}\left(\frac{t+2}{3}\right) + c$$

$$= \sin^{-1}\left(\frac{e^x + 2}{3}\right) + c$$

9) $\int \frac{\cos x}{\sqrt{4 - \sin^2 x}}\ dx$

Ans. $I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}}\ dx$

Put $\sin x = t$

$\therefore$ $\cos x\,dx = dt$

$\therefore$ $I = \int \frac{dt}{\sqrt{4 - t^2}}$

$$= \int \frac{dt}{\sqrt{(2)^2 - (t)^2}}$$

$$= \sin^{-1}\left(\frac{t}{2}\right) + c$$

$$= \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

10) $\int \frac{\cos x}{\sqrt{7 - \sin^2 x - 4\sin x}}\ dx$

Ans. $I = \int \frac{\cos x}{\sqrt{7 - \sin^2 x - 4\sin x}} dx$

put $\sin x = t$

$\therefore$ cos x dx = dt

$$\therefore \quad I = \int \frac{dt}{\sqrt{7 - t^2 - 4t}}$$

$$= \int \frac{dt}{\sqrt{7 - \left(t^2 + 4t + 4\right) + 4}}$$

$$= \int \frac{dt}{\sqrt{\left(\sqrt{11}\right)^2 - (t+2)^2}}$$

$$= \sin^{-1}\left(\frac{t+2}{\sqrt{11}}\right) + c$$

$$= \sin^{-1}\left(\frac{\sin x + 2}{\sqrt{11}}\right) + c$$

11) $\int \frac{\sec^2 x \ dx}{\sqrt{1 - \tan^2 x}}$

Ans. $I = \int \frac{\sec^2 x \ dx}{\sqrt{1 - \tan^2 x}}$

put tan x = t

$\therefore$ $\sec^2 x$ dx = dt

$$\therefore \quad I = \int \frac{dt}{\sqrt{1 - t^2}}$$

$= \sin^{-1}$ (t) + c

$= \sin^{-1}$(tan x) + c

12) $\int \frac{\cos x}{\sqrt{9 + 8 \sin x - \sin^2 x}} dx$

Ans. Let sin x = t

$\therefore$ cos x dx = dt

$$\therefore \quad I = \int \frac{dt}{\sqrt{-(t^2 - 8t + 16) + 16 + 9}}$$

$$= \int \frac{dt}{\sqrt{5^2 - (t-4)^2}}$$

$$= \sin^{-1}\left(\frac{t-4}{5}\right) + c$$

$$= \sin^{-1}\left(\frac{\sin x - 4}{5}\right) + c$$

13) $\int \frac{dx}{x\sqrt{\left(\log x^2\right) + 4}} dx$

Ans. $I = \int \frac{dx}{x\sqrt{\left(\log x^2\right) + 4}} dx$

let log x = t

$\frac{1}{x} = dx = dt$

$$\int \frac{dt}{\sqrt{t^2 + 2^2}}$$

$$\log\left|t + \sqrt{t^2 + 2^2}\right| + c$$

$$\log\left|\log x + \sqrt{(\log x)^2 + 4}\right| + c$$

14) $\int \frac{\sec^2 x \, dx}{\sqrt{1 - \tan^2 x}}$

Ans. Let, $I = \int \frac{\sec^2 x \, dx}{\sqrt{1 - \tan^2 x}}$

Put tan x = t

$\therefore$ $\sec^2 x$ dx = dt

$$I = \int \frac{dt}{\sqrt{1 - t^2}}$$

$= \sin^{-1}$ (t)

$= \sin^{-1}$(tan x) + c

GROUP (L)-HOME WORK PROBLEMS

1) $$\int \frac{5x + 2}{\sqrt{3x^2 + 4x + 5}}\,dx$$

Ans. $$I = \int \frac{5x + 2}{\sqrt{3x^2 + 4x + 5}}\,dx$$

let $5x + 2 = A\,\frac{d}{dx}\,(3x^2 + 4x + 5) + B$

∴ $5x + 2 = A\,(6x + 4) + B$ ... (i)

∴ $5x + 2 = 6Ax + 4A + B$

So $6A = 5$

$A = \frac{5}{6}$

Also, $4A + B = 2$

∴ $4\left(\frac{5}{6}\right) + B = 2$

∴ $\frac{10}{3} + B = 2$

∴ $B = 2 - \frac{10}{3} = \frac{-4}{3}$

$$\text{So, } I = \int \frac{\left(\frac{5}{6}(6x+4) - \frac{4}{3}\right)}{\sqrt{3x^2 + 4x + 5}}\,dx$$

$$= \frac{5}{6}\int \frac{6x+4}{\sqrt{3x^2+4x+5}}\,dx - \frac{4}{3}\int \frac{1}{\sqrt{3x^2+4x+5}}\,dx$$

$$= \frac{5}{6} \times 2\sqrt{3x^2+4x+5} - \frac{4}{3}\int \frac{1}{\sqrt{3\left(x^2 + \frac{4x}{3} + \frac{5}{3}\right)}}$$

$$= \frac{5}{3} \times \sqrt{3x^2+4x+5} - \frac{4}{3\sqrt{3}}\int \frac{1}{\sqrt{x^2 + \frac{4x}{3} + \frac{4}{9} + \frac{5}{3} - \frac{4}{9}}}\,dx$$

$$= \frac{5}{3} \times \sqrt{3x^2+4x+5} - \frac{4}{3\sqrt{3}}\int \frac{1}{\sqrt{\left(x + \frac{2}{3}\right)^2 + \left(\frac{\sqrt{11}}{3}\right)^2}}\,dx$$

$$= \frac{5}{3} \times \sqrt{3x^2+4x+5}$$

$$- \frac{4}{3\sqrt{3}}\log\left|x + \frac{2}{3} + \sqrt{x^2 + \frac{4x}{3} + \frac{5}{3}}\right| + c$$

2) $$\int \frac{2x - 5}{\sqrt{x^2 - 4x + 5}}\,dx$$

Ans. Let $2x - 5 = A\,\frac{d}{dx}(x^2 - 4x + 5) + B$

∴ $2x - 5 = A\,(2x - 4) + B$

∴ $2x - 5 = 2Ax - 4A + B$

So, $2A = 2$

∴ $A = 1$

Also, $- 4A + B = - 5$

∴ $B = - 5 + 4 = - 1$

$$\text{So, } I = \int \frac{(1(2x-4)-1)}{\sqrt{x^2-4x+5}}\,dx$$

$$= \int \frac{(2x-4)}{\sqrt{x^2-4x+5}}\,dx - \int \frac{1}{\sqrt{x^2-4x+5}}\,dx$$

$$= 2\sqrt{x^2-4x+5} - \int \frac{1}{\sqrt{x^2-4x+4+5-4}}\,dx$$

$$= 2\sqrt{x^2-4x+5} - \int \frac{1}{\sqrt{(x-2)^2 + (1)^2}}\,dx$$

$$= 2\sqrt{x^2-4x+5} - \log\left|x - 2 + \sqrt{x^2-4x+5}\right| + c$$

3) $$\int \frac{3x + 1}{\sqrt{3x^2 - 4x - 5}}\,dx$$

Ans. Let $3x + 1 = A\frac{d}{dx}\,(3x^2 - 4x - 5) + B$

∴ $3x + 1 = A\,(6x - 4) + B$

∴ $3x + 1 = 6Ax - 4A + B$

So, $6A = 3$

∴ $A = \frac{1}{2}$

Also, $- 4A + B = 1$

∴ $- 4\left(\frac{1}{2}\right) + B = 1$

∴ $B = 3$

$$\text{So, } I = \int \frac{\frac{1}{2}(6x-4) + 3}{\sqrt{3x^2+4x-5}}\,dx$$

$$= \frac{1}{2}\int \frac{6x-4}{\sqrt{3x^2-4x-5}}\,dx + \int \frac{3}{\sqrt{3x^2-4x-5}}\,dx$$

$$= \frac{1}{2} \times 2\sqrt{3x^2-4x-5} + \sqrt{3}\int \frac{1}{\sqrt{3\left(x^2 - \frac{4x}{3} + \frac{5}{3}\right)}}\,dx$$

$$= \sqrt{3x^2-4x-5} + \frac{3}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 - \frac{4x}{3} + \frac{4}{9} + \frac{5}{3} - \frac{4}{9}}}\, dx$$

$$= \sqrt{3x^2-4x-5} + \sqrt{3} \int \frac{1}{\sqrt{\left(x-\frac{2}{3}\right)^2 + \left(\frac{\sqrt{11}}{3}\right)^2}}\, dx$$

$$= \sqrt{3x^2-4x-5} + \sqrt{3}\log\left|x - \frac{2}{3} + \sqrt{x^2 - \frac{4x}{3} + \frac{5}{3}}\right| + c$$

4) $\int \sqrt{\frac{x+3}{x+2}}\, dx$

Ans. $I = \int \sqrt{\frac{x+3}{x+2}}\, dx$

$$= \int \sqrt{\frac{x+3}{x+2}} \times \sqrt{\frac{x+3}{x+3}}\, dx$$

$$= \int \frac{x+3}{\sqrt{x^2+5x+6}}\, dx$$

Let $x + 3 = A\frac{d}{dx}(x^2 + 5x + 6) + B$

$\therefore$ $x + 3 = A(2x + 5) + B$

$\therefore$ $2A = 1 \quad \therefore A = \frac{1}{2}$

Also, $5A + B = 3$

$\therefore$ $5\left(\frac{1}{2}\right) + B = 3$

$B = 3 - \frac{5}{2} = \frac{1}{2}$

So, $I = \int \frac{\left(\frac{1}{2}(2x+5) + \frac{1}{2}\right)}{\sqrt{x^2+5x+6}}\, dx$

$$= \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\, dx + \frac{1}{2}\int \frac{1}{\sqrt{x^2+5x+6}}\, dx$$

$$= \frac{1}{2} \times 2\sqrt{x^2+5x+6} + \frac{1}{2}\int \frac{1}{\sqrt{x^2+5x+\frac{25}{4}+6-\frac{25}{4}}}\, dx$$

$$= \sqrt{x^2+5x+6} + \frac{1}{2}\int \frac{dx}{\sqrt{\left(x+\frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$\sqrt{x^2+5x+6} + \frac{1}{2}\log\left|x + \frac{5}{2} + \sqrt{x^2+5x+6}\right| + c$$

5) $\int \sqrt{\frac{x+1}{9-x}}\, dx$

Ans. $I = \int \sqrt{\frac{x+1}{9-x}}\, dx$

$$= \int \sqrt{\frac{x+1}{9-x}} \times \sqrt{\frac{x+1}{x+1}}\, dx$$

$$= \int \frac{x+1}{\sqrt{-x^2+8x+9}}\, dx$$

$x + 1 = A\frac{d}{dx}(-x^2 + 8x + 9) + B$

$\therefore$ $x + 1 = A(-2x + 8) + B$

$\therefore$ $-2A = 1$

$\therefore$ $A = \frac{-1}{2}$

Also, $8A + B = 1$

$\therefore$ $8\left(\frac{-1}{2}\right) + B = 1$

$\therefore$ $B = 5$

So, $I = \int \frac{\frac{-1}{2}(-2x+8)+5}{\sqrt{-x^2+8x+9}}\, dx$

$$= \frac{-1}{2}\int \frac{(-2x+8)}{\sqrt{-x^2+8x+9}} + 5\int \frac{1}{\sqrt{9-\left(x^2-8x\right)}}\, dx$$

$$= \frac{-1}{2} \times 2\sqrt{-x^2+8x+9} + 5\int \frac{1\, dx}{\sqrt{9-\left(x^2-8x+16x\right)+16}}$$

$$= -\sqrt{-x^2+8x+9} + 5\int \frac{1}{\sqrt{(5)^2-(x-4)^2}}\, dx$$

$$= -\sqrt{-x^2+8x+9} + 5\sin^{-1}\left(\frac{x-4}{5}\right) + c$$

6) $\int \sqrt{\frac{x-2}{x}}\, dx$

Ans. $I = \int \frac{x-2}{\sqrt{x^2-2x}}\, dx$

$$= \int \sqrt{\frac{x-2}{x}} \times \sqrt{\frac{x-2}{x-2}}\, dx$$

$$= \int \frac{x-2}{\sqrt{x^2-2x}}\, dx$$

Let $x - 2 = A\frac{d}{dx}(x^2 - 2x) + B$

∴ $x - 2 = A\,(2x - 2) + B$

∴ $2A = 1$

∴ $A = \dfrac{1}{2}$

So, $-\,2A + B = -\,2$

∴ $-\,2\left(\dfrac{1}{2}\right) + B = -\,2$

∴ $B = -1$

$$\text{So, } I = \int \frac{\frac{1}{2}(2x-2)-1}{\sqrt{x^2-2x}}\,dx$$

$$= \frac{1}{2}\int \frac{2x-2}{\sqrt{x^2-2x}}\,dx - \int \frac{1}{\sqrt{x^2-2x}}\,dx$$

$$= \frac{1}{2}\times 2\sqrt{x^2-2x} - \int \frac{1}{\sqrt{x^2-2x+1-1}}\,dx$$

$$= \sqrt{x^2-2x} - \int \frac{1}{\sqrt{(x-1)^2+(1)^2}}\,dx$$

$$= \sqrt{x^2-2x} - \log\left|x-1+\sqrt{x^2-2x}\right| + c$$

GROUP (M)-HOME WORK PROBLEMS

1) $\displaystyle\int \frac{1}{7\cos^2 x - 4}\,dx$

Ans. Let, $I = \displaystyle\int \frac{1}{7\cos^2 x - 4}\,dx$

Dividing Nr and Dr by $\cos^2 x$,

∴ $I = \displaystyle\int \frac{\sec^2 x}{7 - 4\sec^2 x}\,dx$

$$= \int \frac{\sec^2 x\,dx}{7 - 4\left(1+\tan^2 x\right)}$$

$$= \int \frac{\sec^2 x\,dx}{3 - 4\tan^2 x}$$

Put $\tan x = t$, ∴ $\sec^2 x\,dx = dt$

∴ $I = \displaystyle\int \frac{dt}{\left(\sqrt{3}\right)^2 - (2t)^2}$

$$= \frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+2t}{\sqrt{3}-2t}\right| \times \frac{1}{2} + c$$

$$= \frac{1}{4\sqrt{3}}\log\left|\frac{\sqrt{3}+2\tan x}{\sqrt{3}-2\tan x}\right| + c$$

2) $\displaystyle\int \frac{1}{\sin^2 x + 2\cos^2 x + 3}\,dx$

Ans. Dividing Nr and Dr by $\cos^2 x$,

∴ $I = \displaystyle\int \frac{\sec^2 x}{\tan^2 x + 2 + 3\sec^2 x}\,dx$

$$= \int \frac{\sec^2 x\,dx}{\tan^2 x + 2 + 3\left(1+\tan^2 x\right)}$$

$$= \int \frac{\sec^2 x}{2 + 3 + 3\tan^2 x + \tan^2 x}\,dx$$

$$= \int \frac{\sec^2 x}{5 + 4\tan^2 x}\,dx$$

Let $\tan x = t$

∴ $\sec^2 x\,dx = dt$

∴ $I = \displaystyle\int \frac{dt}{(\sqrt{5})^2 + (2t)^2}$

∴ $I = \dfrac{1}{\sqrt{5}}\tan^{-1}\left(\dfrac{2t}{\sqrt{5}}\right)\dfrac{1}{2} + c$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c.$$

3) $\displaystyle\int \frac{1}{2\sin^2 x - 3\cos^2 x + 7}\,dx$

Ans. $I = \displaystyle\int \frac{1}{2\sin^2 x - 3\cos^2 x + 7}\,dx$

Dividing Nr and Dr by $\cos^2 x$

∴ $I = \displaystyle\int \frac{\sec^2 x}{2\tan^2 x - 3 + 7\sec^2 x}\,dx$

$$= \int \frac{\sec^2 x}{2\tan^2 x - 3 + 7\left(1+\tan^2 x\right)}\,dx$$

$$= \int \frac{\sec^2 x\,dx}{9\tan^2 x + 4}$$

put $\tan x = t$

∴ $\sec^2 x\,dx = dt$

∴ $I = \displaystyle\int \frac{dt}{9t^2 + 4}$

$$= \int \frac{dt}{(3t)^2 + (2)^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{3t}{2}\right)\times\frac{1}{3}+c$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)+c$$

4) $\int \frac{1}{4\cos^2 x+\sin^2 x}dx$

Ans. $I = \int \frac{1}{4\cos^2 x+\sin^2 x}dx$

Dividing Nr and Dr by $\cos^2 x$

$\therefore \quad I = \int \frac{\sec^2 x}{4+\tan^2 x}\,dx$

put tan x = t

$\therefore \quad I = \int \frac{dt}{(t)^2+(2)^2}$

$$= \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+c$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{\tan x}{2}\right)+c$$

5) $\int \frac{1}{3-2\sin^2 x}\,dx$

Ans. $I = \int \frac{1}{3-2\sin^2 x}\,dx$

$$= \int \frac{\sec^2 x\,dx}{3\sec^2 x-2\tan^2 x}$$

$$= \int \frac{\sec^2 x\,dx}{3\left(1+\tan^2 x\right)-2\tan^2 x}$$

$$= \int \frac{\sec^2 x\,dx}{3+\tan^2 x}$$

put tan x = t

$\therefore \quad \sec^2 x\,dx = dt$

$\therefore \quad I = \int \frac{dt}{3+t^2}$

$$= \int \frac{dt}{(t)^2+\left(\sqrt{3}\right)^2}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)+c$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\tan x}{3}\right)+c$$

GROUP (N)-HOME WORK PROBLEMS

1) $\int \frac{1}{4-5\sin x}$

Ans. Let $\tan\frac{x}{2} = t$

$\frac{x}{2} = \tan^{-1} t$

$x = 2\tan^{-1} t$

$\therefore \quad dx = \frac{2}{1+t^2}\,dt$

$\sin x = \frac{2t}{1+t^2}$

$\therefore \quad \int \frac{1}{4-5\sin x}\,dx$

So,

$$I = \int \frac{\frac{2\,dt}{1+t^2}}{4-5\left(\frac{2t}{1+t^2}\right)}$$

$$= 2\int \frac{1}{4+4t^2-10t}\,dt$$

$$= \frac{2}{4}\int \frac{1}{t^2-\frac{5}{2}t+1}\,dt$$

$$= \frac{1}{2}\int \frac{1}{t^2-\frac{5}{2}t+\frac{25}{16}-\frac{25}{16}+1}\,dt$$

$$= \frac{1}{2}\int \frac{1}{\left(t-\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}\,dt$$

$$= \frac{1}{2}\times\frac{1}{2\times\frac{3}{4}}\log\left|\frac{t-\frac{5}{4}-\frac{3}{4}}{t-\frac{5}{4}+\frac{3}{4}}\right|+c$$

$$= \frac{1}{3}\log\left|\frac{t-2}{t-\frac{1}{2}}\right|+c$$

$$= \frac{1}{3}\log\left|\frac{2t-4}{2t-1}\right|+c$$

$$= \frac{1}{3}\log\left|\frac{2\tan\left(\frac{x}{2}\right) - 4}{2\tan\left(\frac{x}{2}\right) - 1}\right| + c$$

2) $\int \frac{1}{5 - 4\sin x}\, dx$

Ans. put $\tan\frac{x}{2} = t$

$\therefore \quad dx = \frac{2dt}{1+t^2}$

$\sin x = \frac{2t}{1+t^2}$

$$\therefore \quad I = \int \frac{\frac{2\,dt}{1+t^2}}{5 - 4\left(\frac{2t}{1+t^2}\right)}$$

$$= \int \frac{2}{5 + 5t^2 - 8t}\, dt$$

$$= \frac{2}{5}\int \frac{1}{t^2 - \frac{8}{5}t + \frac{16}{25} + 1 - \frac{16}{25}}$$

$$= \frac{2}{5}\int \frac{1}{\left(t - \frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2}\, dt$$

$$= \frac{2}{5}\,\frac{1}{\frac{3}{5}}\tan^{-1}\left(\frac{t - \frac{4}{5}}{\frac{3}{5}}\right) + c$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{5t - 4}{3}\right) + c$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{5\tan(x/2) - 4}{3}\right) + c.$$

3) $\int \frac{1}{4 + 5\cos x}$

Ans. put $\tan\frac{x}{2} = t$

$\therefore \quad dx = \frac{2dt}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$$\therefore \quad I = \int \frac{\frac{2\,dt}{1+t^2}}{4 + 5\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= 2\int \frac{dt}{4 + 4t^2 + 5 - 5t^2}$$

$$= 2\int \frac{1}{-t^2 + 9}\, dt$$

$$= 2\int \frac{1}{-t^2 + 3^2}\, dt$$

$$= 2 \times \frac{1}{2 \times 3}\log\left|\frac{3+t}{3-t}\right| + c$$

$$= \frac{1}{3}\log\left|\frac{3 + \tan(x/2)}{3 - \tan(x/2)}\right| + c$$

4) $\int \frac{1}{\sqrt{x} - \sqrt[3]{x}}\, dx$

Ans. $I = \int \frac{1}{\sqrt{x} - \sqrt[3]{x}}\, dx$

$$= \int \frac{1}{x^{\frac{1}{2}} - x^{\frac{1}{3}}}\, dx$$

put $x = t^6 \quad \therefore t = x^{\frac{1}{6}}$

$\therefore \quad dx = 6t^5\, dt$

So,

$$I = \int \frac{6t^5 dt}{\left(t^6\right)^{\frac{1}{2}} - \left(t^6\right)^{\frac{1}{3}}}$$

$$= \int \frac{6t^5}{t^3 - t^2}\, dt$$

$$= \int \frac{6t^5}{t^2(t-1)}\, dt$$

$$= \int \frac{6t^3}{t-1}\, dt$$

$$= 6\int \frac{t^3 - 1 + 1}{t-1}\, dt$$

$$= 6\int \left[\frac{(t-1)\left(t^2 + t + 1\right)}{(t-1)} + \frac{1}{t-1}\right] dt$$

$$= 6\int\left(t^2+t+1+\frac{1}{t-1}\right)dt$$

$$= 6\left[\frac{t^3}{3}+\frac{t^2}{2}+t+\log\left|t-1\right|\right]+c$$

$$= 6\left[\frac{\left(x^{\frac{1}{6}}\right)^3}{3}+\frac{\left(x^{\frac{1}{6}}\right)^3}{2}+(x)^{\frac{1}{6}}+\log\left|x^{\frac{1}{6}}-1\right|\right]+c$$

$$= 6\left[\frac{x^{\frac{1}{2}}}{3}+\frac{x^{\frac{1}{3}}}{2}+x^{\frac{1}{6}}+\log\left|x^{\frac{1}{6}}-1\right|\right]+c$$

5) $\int\frac{\sqrt{x}}{1+\sqrt[4]{x^3}}\,dx$

Ans. $I = \int\frac{\sqrt{x}}{1+\sqrt[4]{x^3}}\,dx$

$$= \int\frac{x^{1/2}}{1+x^{3/4}}\,dx$$

put $x = t^4, t = x^{\frac{1}{4}}$

$\therefore$ $dx = 4t^3\ dt$

So,

$$I = \int\frac{t^2}{1+\left(t^4\right)^{\frac{3}{4}}}\times 4t^3\ dt$$

$$= \int\frac{4t^5}{1+t^3}\ dt$$

$$= \int\frac{4t^3\ t^2\ dt}{1+t^3}$$

put $1 + t^3 = m$ Also, $t^3 = m - 1$

$\therefore$ $3t^2\ dt = dm \quad t^2\ dt = \frac{dm}{3}$

So,

$$I = \int\frac{4(m-1)}{m}\,\frac{dm}{3}$$

$$= \frac{4}{3}\left(m-\log\left|m\right|\right)+c$$

$$= \frac{4}{3}\left(1+t^3-\log\left|1+t^3\right|\right)+c$$

$$= \frac{4}{3}\left(1+\left(x^{\frac{1}{4}}\right)^3-\log\left|1+\left(x^{\frac{1}{4}}\right)^3\right|\right)+c$$

$$= \frac{4}{3}\left(1+x^{\frac{3}{4}}-\log\left|1+\sqrt[4]{x^3}\right|\right)+c$$

6) $\int\frac{1}{4+5\sin x}\,dx$

Ans. $I= \int\frac{\frac{2dt}{1+t^2}}{4+5\left(\frac{2t}{1+t^2}\right)}$

$$= 2\int\frac{dt}{4+4t^2+10t}$$

$$= \frac{2}{4}\int\frac{dt}{t^2+\frac{5}{2}t+1}$$

$$= \frac{1}{2}\int\frac{dt}{t^2+\frac{5}{2}t+\frac{25}{16}-\frac{25}{16}+1}$$

$$= \frac{1}{2}\int\frac{dt}{\left(t+\frac{5}{4}\right)^2-\left(\frac{3}{4}\right)^2}$$

$$= \frac{1}{2}\times\frac{1}{2\times\frac{3}{4}}\log\left|\frac{t+\frac{5}{4}-\frac{3}{4}}{t+\frac{5}{4}+\frac{3}{4}}\right|$$

$$= \frac{1}{3}\log\left|\frac{2t-1}{2t+4}\right|$$

$$= \frac{1}{3}\log\left|\frac{2\tan\left(\frac{x}{2}\right)-1}{2\tan\left(\frac{x}{2}\right)+4}\right|+c$$

7) $$\int \frac{1}{13 + 3\cos x + 4\sin x}\, dx$$

Ans. $$I= \int \frac{\frac{2}{1 + t^2}dt}{13 + 3\left(\frac{1 - t^2}{1 + t^2}\right) + 4\left(\frac{2t}{1 + t^2}\right)}$$

$$= 2 \int \frac{1}{13 + 13t^2 + 3 - 3t^2 + 8t}\, dt$$

$$= 2 \int \frac{1}{10t^2 + 8t + 16}\, dt$$

$$= \frac{2}{10} \int \frac{1}{t^2 + \frac{4}{5}t + \frac{8}{5}}\, dt$$

$$= \frac{1}{5} \int \frac{dt}{t^2 + \frac{4t}{5} + \frac{4}{25} + \frac{8}{5} - \frac{4}{25}}$$

$$= \frac{1}{5} \int \frac{1}{\left(t + \frac{2}{5}\right)^2 + \left(\frac{6}{5}\right)^2}\, dt$$

$$= \frac{1}{5 \times \frac{6}{5}} \tan^{-1}\left(\frac{t + \frac{2}{5}}{\frac{6}{5}}\right) + c$$

$$= \frac{1}{6} \tan^{-1}\left(\frac{5t + 2}{6}\right) + c$$

$$= \frac{1}{6} \tan^{-1}\left(\frac{2 + 5\tan\left(\frac{x}{2}\right)}{6}\right) + c$$

8) $$\int \frac{1}{3(1 - \sin x) - \cos x}\, dx$$

Ans. $$= \int \frac{1}{3 - 3\sin x - \cos x}\, dx$$

$$= 2 \int \frac{\frac{dt}{1 + t^2}}{3 - 3\left(\frac{2t}{1 + t^2}\right) - \left(\frac{1 - t^2}{1 + t^2}\right)}$$

$$= 2 \int \frac{1}{3 + 3t^2 - 6t - 1 + t^2}\, dt$$

$$= 2 \int \frac{1}{4t^2 - 6t + 2}\, dt$$

$$= \frac{2}{2} \times \frac{1}{2} \int \frac{1}{t^2 - \frac{3}{2}t + \frac{1}{2}}\, dt$$

$$= \frac{1}{2} \int \frac{dt}{t^2 - \frac{3}{2}t + \frac{9}{16} + \frac{1}{2} - \frac{9}{16}}$$

$$= \frac{1}{2} \int \frac{1}{\left(t - \frac{3}{4}\right)^2 - \left(\frac{1}{4}\right)^2}\, dt$$

$$= \frac{1}{2 \times 2 \times \frac{1}{4}} \log \left|\frac{t - \frac{3}{4} - \frac{1}{4}}{t - \frac{3}{4} + \frac{1}{4}}\right| + c$$

$$= \log \left|\frac{2t - 2}{2t - 1}\right| + c$$

$$= \log \left|\frac{2\tan\left(\frac{x}{2}\right) - 2}{2\tan\left(\frac{x}{2}\right) - 1}\right| + c$$

9) $$\int \frac{1}{3\sin x + 4\cos x + 5}\, dx$$

Ans. put $\tan\frac{x}{2} = t$

$\therefore$ $dx = \frac{2dt}{1 + t^2}$

$\sin x = \frac{2t}{1 + t^2}$

$\cos x = \frac{1 - t^2}{1 + t^2}$

$\therefore$ $$I = \int \frac{\frac{2\,dt}{1 + t^2}}{3\left(\frac{2t}{1 + t^2}\right) + 4\left(\frac{1 - t^2}{1 + t^2}\right) + 5}$$

$$= \int \frac{2}{6t + 4 - 4t^2 + 5 + 5t^2}\, dt$$

$$= 2 \int \frac{1}{t^2 + 6t + 9}\, dt$$

$$= 2 \int \frac{1}{(t + 3)^2}\, dt$$

$$= \frac{2\,(-1)}{t + 3} + c$$

$$= \frac{-2}{\tan\frac{x}{2} + 3} + c$$

10) $\int \frac{1}{\cos\alpha - \cos x}\,dx$

Ans. $$I = \int \frac{\frac{2dt}{1+t^2}}{\cos\alpha - \left(\frac{1-t^2}{1+t^2}\right)}$$

$$= 2\int \frac{1}{\cos\alpha + t^2\cos\alpha - 1 + t^2}\,dt$$

$$= 2\int \frac{1}{t^2(1+\cos\alpha) - (1-\cos\alpha)}\,dt$$

$$= 2\int \frac{1}{t^2 \cdot 2\cos^2\frac{\alpha}{2} - 2\sin^2\frac{\alpha}{2}}$$

$$= \frac{2}{2}\int \frac{1}{\left(t\cos\frac{\alpha}{2}\right)^2 - \left(\sin\frac{\alpha}{2}\right)^2}\,dt$$

$$= \int \frac{1}{\left(t\cos\frac{\alpha}{2}\right)^2 - \left(\sin\frac{\alpha}{2}\right)^2}\,dt$$

$$= \frac{1}{2\times\sin\frac{\alpha}{2}}\log\left|\frac{t\cos\left(\frac{\alpha}{2}\right) - \sin\frac{\alpha}{2}}{t\cos\left(\frac{\alpha}{2}\right) + \sin\frac{\alpha}{2}}\right| \times$$

$$\frac{1}{\cos\frac{\alpha}{2}} + c$$

$$= \frac{1}{\sin\alpha}\log\left|\frac{\tan\left(\frac{x}{2}\right) - \tan\left(\frac{\alpha}{2}\right)}{\tan\left(\frac{x}{2}\right) + \tan\left(\frac{\alpha}{2}\right)}\right| + c$$

11) $\int \frac{1}{1+\cos\propto.\cos x}\,dx$

Ans. put $\tan\frac{x}{2} = t$

$\therefore$ $dx = \frac{2dt}{1+t^2}$

$$\cos x = \frac{1-t^2}{1+t^2}$$

So,

$$I = \int \frac{\frac{2dt}{1+t^2}}{1+\cos\alpha\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= \int \frac{2\,dt}{1+t^2+\cos\alpha\left(1-t^2\right)}$$

$$= \int \frac{2\,dt}{(1+\cos\alpha)+(1-\cos\alpha)\,t^2}$$

$$= \int \frac{2dt}{\left(2\cos^2\alpha/2\right)+\left(2\sin^2\alpha/2\right)t^2}$$

$$= \int \frac{dt}{\left(\cos\frac{\alpha}{2}\right)^2 + \left(t\sin\frac{\alpha}{2}\right)^2}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\tan^{-1}\left(\frac{t\sin\alpha/2}{\cos\alpha/2}\right)\times\frac{1}{\sin\frac{\alpha}{2}} + c$$

$$= \frac{2}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\tan^{-1}\left(\tan\frac{x}{2}\tan\frac{\alpha}{2}\right) + c$$

$$= \frac{2}{\sin\alpha}\tan^{-1}\left(\tan\frac{x}{2}\tan\frac{\alpha}{2}\right) + c$$

$$= 2\operatorname{cosec}\alpha\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\alpha}{2}\right) + c$$

12) $\int \frac{1+\sin x\sin\alpha}{\sin\alpha + \sin x}\,dx$

Ans. Writing $1 = \sin^2\alpha + \cos^2\alpha$

$$I = \int \frac{\sin^2\alpha + \sin x\sin\alpha + \cos^2\alpha}{\sin\alpha + \sin\alpha}\,dx$$

$$= \int \frac{\sin\alpha(\sin\alpha + \sin x) + \cos^2\alpha}{\sin\alpha + \sin x}\,dx$$

$$= \int\left(\sin\alpha + \frac{\cos^2\alpha}{\sin\alpha + \sin x}\right)dx$$

$$= \sin\alpha\int 1\,dx + \cos^2\alpha\int\frac{dx}{\sin\alpha + \sin x}$$

$= x\sin\alpha + (\cos^2\alpha)I_1$ ....(say)

In I_1, put tan (x/2)= t. Then x = 2 $\tan^{-1}$ t.

$$\therefore \quad dx = \frac{2dt}{1+t^2} \text{ and } \therefore \sin x = \frac{2t}{1+t^2}$$

$$\therefore \quad I_1 = \int \frac{1}{\sin\alpha + \frac{2t}{1+t^2}} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{2dt}{\left(1+t^2\right)\sin\alpha + 2t} = \frac{2}{\sin\alpha}\int \frac{dt}{t^2 + 2t\operatorname{cosec}\alpha + 1}$$

$$= \frac{2}{\sin\alpha}\int \frac{dt}{\left(t^2 + 2t\operatorname{cosec}\alpha + \operatorname{cosec}^2 x\right) - \left(\operatorname{cosec}^2\alpha - 1\right)}$$

$$= \frac{2}{\sin\alpha}\int \frac{dt}{(t+\operatorname{cosec}\alpha)^2 - \cot^2\alpha}$$

$$= \frac{2}{\sin\alpha} \cdot \frac{1}{2\cot\alpha}\log\left|\frac{t+\operatorname{cosec}\alpha - \cot\alpha}{t+\operatorname{cosec}\alpha + \cot\alpha}\right|$$

$$= \frac{1}{\cos\alpha}\log\left|\frac{\tan(x/2)+\operatorname{cosec}\alpha - \cot\alpha}{\tan(x/2)+\operatorname{cosec}\alpha + \cot\alpha}\right| + c$$

$\therefore$ I = x sin α +

$$\frac{\cos^2\alpha}{\cos\alpha}\log\left|\frac{\tan(x/2)+\operatorname{cosec}\alpha - \cot\alpha}{\tan(x/2)+\operatorname{cosec}\alpha + \cot\alpha}\right| + c$$

= x sin α +

$$(\cos\alpha)\log\left|\frac{\tan(x/2)+\operatorname{cosec}\alpha - \cot\alpha}{\tan(x/2)+\operatorname{cosec}\alpha + \cot\alpha}\right| + c$$

13) $\int \frac{1}{2 - 3\sin 2x}\,dx$

Ans. tan x = t

$\therefore$ x = $\tan^{-1}$ t

$$dx = \frac{1}{1+t^2}\,dt$$

$$\therefore \quad \int \frac{\frac{1}{1+t^2}}{2 - 3\left(\frac{2t}{1+t^2}\right)}\,dt$$

$$= \int \frac{1}{2 - 2t^2 - 6t}\,dt$$

$$= \frac{1}{2}\int \frac{1}{t2 - 3t + 1}\,dt$$

$$= \frac{1}{2}\int \frac{1}{t^2 - 3t + \frac{9}{4} - \frac{9}{4} + 1}\,dt$$

$$= \frac{1}{2}\int \frac{1}{\left(t - \frac{3}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2}\,dt$$

$$= \frac{1}{2} \times \frac{1}{2 \times \frac{\sqrt{5}}{2}}\log\left|\frac{t - \frac{3}{2} - \frac{\sqrt{5}}{2}}{t - \frac{3}{2} + \frac{\sqrt{5}}{2}}\right| + c$$

$$= \frac{1}{2\sqrt{5}}\log\left|\frac{2\tan x - 3 - \sqrt{5}}{2\tan x - 3 + \sqrt{5}}\right| + c$$

14) $\int \frac{1}{3\cos 2x - 2}\,dx$

Ans. Let $I = \int \frac{1}{3\cos 2x - 2}\,dx$

$$= \int \frac{1}{3\left(2\cos^2 x - 1\right) - 2}\,dx$$

$$= \int \frac{1}{6\cos^2 x - 5}\,dx$$

$$= \int \frac{\sec^2 x}{6 - 5\sec^2 x}\,dx$$

$$= \int \frac{\sec^2 x}{6 - 5\left(1 + \tan^2 x\right)}\,dx$$

$$= \int \frac{\sec^2 x}{1 - 5\tan^2 x}\,dx$$

put tan x = t

$\therefore$ $\sec^2 x$ d x = dt

$$\therefore \quad I = \int \frac{dt}{1 - 5t^2}\,dt$$

$$= \frac{1}{5}\int \frac{1}{\frac{1}{5} - t^2}\,dt$$

$$= \frac{1}{5}\int \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2 - (t)^2}\,dt$$

$$= \frac{1}{5} \times \frac{1}{2 \times \frac{1}{\sqrt{5}}}\log\left|\frac{\frac{1}{\sqrt{5}} + t}{\frac{1}{\sqrt{5}} - t}\right| + c$$

$$= \frac{1}{2\times\sqrt{5}}\log\left|\frac{1+\sqrt{5}t}{1-\sqrt{5}t}\right| + c$$

$$= \frac{1}{2\sqrt{5}}\log\left|\frac{1+\sqrt{5}\tan x}{1-\sqrt{5}\tan x}\right| + c$$

15) $\int \frac{1}{3 - 2\sin x}\, dx$

Ans. $= \int \frac{\frac{2}{1+t^2}}{3 - 2\left(\frac{2t}{1+t^2}\right)}\, dt$

$$= 2\int \frac{1}{3 + 3t^2 - 4t}\, dt$$

$$= \frac{2}{3}\int \frac{1}{t^2 - \frac{4}{3}t + 1}\, dt$$

$$= \frac{2}{3}\int \frac{1}{t^2 - \frac{4}{3}t + \frac{4}{9} - \frac{4}{9} + 1}\, dt$$

$$= \frac{2}{3}\int \frac{1}{\left(t - \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2}\, dt$$

$$= \frac{2}{3 \times \frac{\sqrt{5}}{3}} \tan^{-1}\left(\frac{t - \frac{2}{3}}{\frac{\sqrt{5}}{3}}\right) + c$$

$$= \frac{2}{\sqrt{5}} \tan^{-1}\left(\frac{3\tan\frac{x}{2} - 2}{\sqrt{5}}\right) + c$$

$$\int \frac{\sec^2 x}{3 + 3\tan^2 x - 2\tan^2 x}\, dx$$

$$= \int \frac{\sec^2 x}{\tan^2 x + 3}\, dx$$

$$= \frac{dt}{t2 + (\sqrt{3})^2}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x}{\sqrt{3}}\right) + c$$

16) $\int \frac{1}{2 - 3\cos^2 x}\, dx$

Ans. $= \int \frac{\sec^2 x}{2\sec^2 x - 3}\, dx$

$$= \int \frac{\sec^2 x}{2 + 2\tan^2 x - 3}\, dx$$

$$= \int \frac{\sec^2 x}{2\tan^2 x - 1}\, dx$$

Let tan x = t

$\therefore$ $\sec^2 x\, dx = dt$

$\therefore$ $I = \int \frac{dt}{2t^2 - 1}$

$$= \int \frac{dt}{(\sqrt{2}t)^2 - 1^2}$$

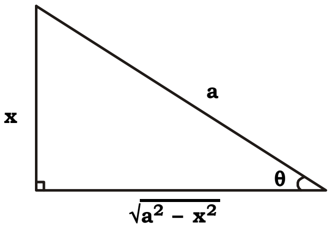
$$= \frac{1}{2 \times 1} \times \frac{1}{\sqrt{2}} \log\left|\frac{\sqrt{2}t - 1}{\sqrt{2}t + 1}\right| + c$$

$$= \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}\tan - 1}{\sqrt{2}\tan + 1}\right| + c.$$

GROUP (O)-HOME WORK PROBLEMS

1) $\int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx$

Ans.



Put x = a sinθ

dx = a cosθ dθ

$$\int \frac{a^2\sin^2\theta\ a\cos\theta}{\sqrt{a^2\ (1 - \sin^2\theta)}}\, d\theta$$

$$= \int \frac{a^2\sin^2\theta}{a\ \cos\theta}\ (a\cos\theta)\, d\theta$$

$$= a^2 \int \frac{1 - \cos 2\theta}{2}\, d\theta\, d\theta$$

$$= \frac{a^2}{2} \int 1 - \cos 2\theta\ d\theta$$

$$= \frac{a^2}{2}\left[\theta - \frac{\sin 2\theta}{2}\right] + c$$

$$= \frac{a^2}{2}\ [\theta - \sin\theta\cos\theta] + c$$

$$\sin\theta = \frac{x}{a}\ \Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\cos\theta = \frac{\sqrt{a^2 - x^2}}{a}$$

$$= \frac{a^2}{2}\left[\sin^{-1}\ \left(\frac{x}{a}\right) - \frac{x\sqrt{a^2 - x^2}}{a^2}\right] + c$$

2) $\int\sqrt{\frac{a - x}{a + x}}\ dx$

Ans. $I = \int\sqrt{\frac{a - x}{a + x}}\ dx$

$$= \int\frac{\sqrt{a - x}}{\sqrt{a + x}}.\frac{\sqrt{a - x}}{\sqrt{a - x}}dx$$

$$= \int\frac{a - x}{\sqrt{a^2 - x^2}}dx$$

$$= a\int\frac{dx}{\sqrt{a^2 - x^2}} + \frac{1}{2}\int\frac{-2x}{\sqrt{a^2 - x^2}}dx$$

$$= a\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}I_1 \qquad \text{...... (say)}$$

Put $a^2 - x^2 = t$. Then $-2x\ dx = dt$

$$\therefore \quad I_1 = \int\frac{dt}{\sqrt{t}} = \int t^{-1/2}\ dt = \frac{t^{1/2}}{1/2}$$

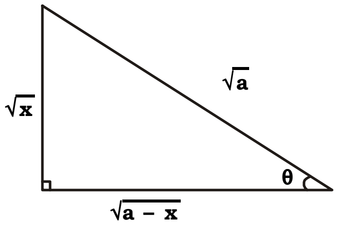
$$= 2\sqrt{a^2 - x^2}$$

$$\therefore \quad I = a\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}.2\sqrt{a^2 - x^2} + c$$

$$= a\sin^{-1}\left(\frac{x}{a}\right) + \sqrt{a^2 - x^2} + c$$

3) $\int\sqrt{\frac{x}{a - x}}\ dx$

Ans.



Put $x = a\sin^2\theta$

$dx = 2a\sin\theta\cos\theta\ d\theta$

$$\theta = \sin^{-1}\left(\frac{\sqrt{x}}{a}\right)$$

$$I = \int\sqrt{\frac{a\sin^2\theta}{a\cos^2\theta}}\ 2a\sin\theta\cos\theta\ d\theta$$

$$= 2a\int\tan\theta\sin\theta\cos\theta\ d\theta$$

$$= 2a\int\sin^2\theta\ d\theta$$

$$= 2a\int\frac{1 - \cos2\theta}{2}\ d\theta$$

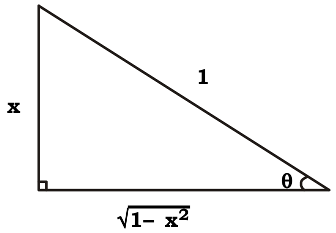
$$= a\int\left[\theta - \frac{\sin2\theta}{2}\right] + c$$

$$= a\ [\theta - \sin\theta\cos\theta] + c$$

$$= a\left(\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right) - \frac{\sqrt{x}\ \sqrt{a - x}}{a}\right) + c$$

4) $\int\frac{x^3}{\sqrt{1 - x^2}}\ dx$

Ans.



Put $x = \sin\theta$ $\quad dx = \cos\theta\ d\theta$

$\theta = \sin^{-1}x$

$$I = \int\frac{\sin^3\theta}{\sqrt{1 - \sin^2\theta}}\ \cos\theta\ d\theta$$

$$\int\frac{\sin^3\theta}{\cos\theta}\ \cos\theta\ d\theta$$

$$\int\frac{3\sin\theta - \sin3\theta}{4}\ d\theta$$

$$\frac{1}{4}\left(-3\cos\theta + \frac{\cos3\theta}{3}\right) + c$$

$$= \frac{1}{4}\left(-3\cos\theta + \frac{4\cos^3\theta - 3\cos\theta}{3}\right) + c$$

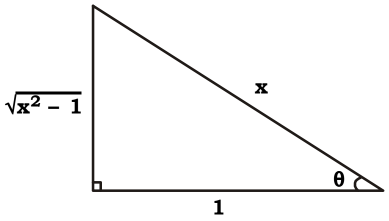
$$= \frac{1}{3 \times 4}\ [-12\cos\theta + 4\cos^3\theta] + c$$

$$= -\cos\theta + \frac{\cos^3\theta}{3} + c$$

$$= -\sqrt{1-x^2} + \frac{(\sqrt{1-x^2})^3}{3} + c$$

5) $\int \frac{1}{x^2\sqrt{x^2-1}}\,dx$

Ans.



Let x = secθ $\quad\therefore\ \theta = \sec^{-1}x$

dx = secθ tanθ dθ

$$\int \frac{\sec\theta\,\tan\theta}{\sec^2\theta\cdot\tan\theta}\,d\theta$$

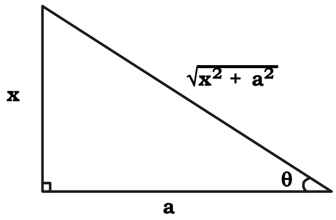
$$= \int \cos\theta\cdot d\theta$$

$$= \sin\theta + c$$

$$= \frac{\sqrt{x^2-1}}{x} + c$$

6) $\int \frac{1}{(x^2+a^2)^2}\,dx$

Ans.



Put x = a tanθ $\quad dx = a\sec^2\theta\,d\theta$

$$\theta = \tan^{-1}\frac{x}{a}$$

$$\therefore\quad I = \int \frac{a\sec^2\theta}{(a^2\cdot\sec^2\theta)^2}\,d\theta$$

$$= \frac{1}{a^3}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{a^3}\int\left[\frac{1+\cos 2\theta}{2}\right]d\theta$$

$$= \frac{1}{2a^3}\left[\theta + \frac{\sin 2\theta}{2}\right] + c$$

$$= \frac{1}{2a^3}[\theta + \sin\theta\cos\theta] + c$$

$$= \frac{1}{2a^3}\left[\tan^{-1}\frac{x}{a} + \frac{ax}{x^2+a^2}\right] + c$$

GROUP (P)-HOME WORK PROBLEMS

1) $\int x\sec^2 x\,dx$

Ans. $= x\int \sec^2 x\,dx - \iint \sec^2 x\,dx\,\frac{d}{dx}(x)\,dx$

$$= x\tan x - \int \tan x\cdot 1\,dx + c_1$$

$$= x\tan x - \log|\sec x| + c$$

2) $\int x\cos nx\,dx$

$$= x\int \cos nx\,dx - \int\left[\int \cos nx\,dx\cdot\frac{d}{dx}(x)\right]dx$$

$$= \frac{x\sin nx}{n} - \int \frac{\sin nx}{n}\,1\,dx$$

$$= \frac{x\sin nx}{n} + \frac{\cos nx}{n^2} + c$$

3) $\int x\tan^2 x\,dx$

Ans. $= x\int \tan^2 x\,dx - \int\left[\int \tan^2 x\,dx\,\frac{d}{dx}(x)\right]dx$

$$= x\int (\sec^2 x - 1)\,dx - \int\left[\int (\sec^2 x - 1)\,dx\right]dx$$

$$= x(\tan x - x) - \int (\tan x - x)\,dx$$

$$= x\tan x - x^2 - \log|\sec x| + \frac{x^2}{2}$$

$$= x\tan x - \log|\sec x| - \frac{x^2}{2} + c$$

4) $\int \log x\cdot x\,dx$

Ans. $= \log x\int x\,dx - \int\int x\,dx\,\frac{d}{dx}(\log x)\,dx$

$$= \frac{x^2}{2}\log x - \int \frac{x^2}{2}\times\frac{1}{x}\,dx + c_1$$

$$= \frac{x^2}{2}\log x - \frac{x^2}{4} + c$$

5) $\int x \sin x \cos 2x \, dx$

Ans. $= \frac{1}{2}\int x 2 \cos 2x \sin x$

$= \frac{1}{2}\int x[\sin 3x - \sin x]$

$= \frac{1}{2}\left[\int (x \sin 3 x - x \sin x)\right] dx$

$$= \frac{1}{2}\left\{\begin{matrix}\left[x \frac{(\cos 3x)}{3} - \int \frac{\cos 3x}{3} dx\right] - \\ \left[-x \cos x - \int -\cos x \, dx\right]\end{matrix}\right\}$$

$= \frac{x}{2} \cos x - \frac{x}{6} \cos 3x - \frac{\sin 3x}{18} - \frac{\sin x}{2} + c$

6) $\int x^2 e^{-2x} dx$

Ans. $= x^2 \int e^{-2x} dx - \int\int e^{-2x} dx \frac{d}{dx} (x^2) dx$

$= x^2 \left(\frac{e^{-2x}}{-2}\right) - \int \frac{e^{-2x}}{-2} (2x) dx + c,$

$= - \frac{x^2 e^{-2x}}{2} +$

$\left[x \int e^{-2x} dx - \int\int e^{-2x} \frac{d}{dx} (x) dx\right] + c_1$

$= - \frac{x^2 e^{-2x}}{2} + \frac{x e^{-2x}}{-2}$

$- \int \frac{e - 2x}{-2} dx + c$

$= - \frac{x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} + c$

$= \left(\frac{-1}{4}\right) e^{-2x} (2x^2 + 2x + 1) + c$

7) $\int x^2 \sin^{-1}x \, dx$

Ans. $= \int \sin^{-1}x \cdot x^2 \, dx$

$= \sin^{-1}x \int x^2 dx - \int\int x^2 \, dx \frac{d}{dx} (\sin^{-1}x) dx$

$= \frac{x^3}{3} \sin^{-1}x - \int \frac{x^3}{3} \frac{1}{\sqrt{1 - x^2}} dx$

$= \frac{1}{3} x^3 \sin^{-1}x - \int \frac{x^3}{\sqrt{1 - x^2}} dx$

Put $(1 - x^2) = t$

∴ $-2x \, dx = dt \;\&\; x^2 = 1 - t$

$= \frac{1}{3} \left(x^3 \sin^{-1} x + \frac{1}{2} \int \frac{1 - t}{\sqrt{t}} dt\right) + c_1$

$= \frac{1}{3} \left(x^3 \sin^{-1} x + \frac{1}{2} \left(\frac{\sqrt{t}}{1/2} - \frac{t^{3/2}}{3/2}\right)\right) + c$

$= \left(x^3 \sin^{-1} x + \sqrt{t} - \frac{t}{3}\right)^{3/2} + c$

$= \frac{x^3 \sin^{-1}x}{3} + \frac{\sqrt{1 - x^2}}{3} - \frac{(1 - x^2)^{3/2}}{9} + c$

8) $\int x^2 \cos^{-1} x \, dx$

Ans. $= \cos^{-1}x \int x^2 \, dx - \int\int x^2 \, dx \frac{d}{dx} (\cos^{-1}x) dx$

$= \frac{x^3 \cos^{-1}x}{3} - \int \frac{x^3}{3} \times \frac{-1}{\sqrt{1 - x^2}} dx$

$= \frac{x^3 \cos^{-1}x}{3} + \frac{1}{3} \int \frac{x^3}{\sqrt{1 - x^2}} dx$

$= \frac{x^3 \cos^{-1}x}{3} - \frac{\sqrt{1 - x^2}}{3} + \frac{(1 - x^2)^{3/2}}{9} + c$

[from earlier solution]

9) $\int x \tan^{-1} x^2 \, dx$

Ans. $= \tan^{-1}x^2 \int x \, dx - \int\left[\int x \, dx \frac{d}{dx} (\tan^{-1} x^2)\right] dx$

$= \frac{x^2}{2} \tan^{-1}x^2 - \int \frac{x^2}{2} \left(\frac{1}{1 + x^4}\right) (2x) dx$

$= \frac{x^2}{2} \tan^{-1}x^2 - 1 \int \frac{x^3}{1 + x^4} dx$

∴ $I = \frac{x^2}{2} \tan^{-1}x^2 - \frac{1}{4} \int \frac{4x^3}{1 + x^4} dx$

$= \frac{x^2}{2} \tan^{-1}x^2 - \frac{1}{4} \log|1 + x^4| + c$

10) $\int \frac{x}{1 - \cos x}\,dx$

Ans. $= \int \frac{x}{2\sin^2 \frac{x}{2}}\,dx$

$$= \frac{1}{2}\int x\,\text{cosec}^2\,\frac{x}{2}\,dx$$

$$= \frac{1}{2}\left[x\int \cos ec^2\,\frac{x}{2} - \int\left(\int \text{cosec}^2\,\frac{x}{2}\,dx\,\frac{d}{dx}(x)\right)\right]dx$$

$$= \frac{1}{2}\left[x\left(-\cot\frac{x}{2}\right)\times 2\right] - \int -\cot\frac{x}{2}\times 2\,dx$$

$$= -x\cot\frac{x}{2} + 2\int \cot\frac{x}{2}\,dx$$

$$= -x\cot\frac{x}{2} + 2\log\left|\sin\frac{x}{2}\right| \times 2 + c$$

$$= -x\cot\frac{x}{2} + 4\log\left|\sin\frac{x}{2}\right| + c$$

11) $\int x^2 \sin x\,dx$

Ans. $= x^2\int \sin x\,dx - \int\left[\int \sin x\,dx\,\frac{d}{dx}(x^2)\right]dx$

$$= -x^2\cos x - \int[-\cos x(2x)]\,dx$$

$$= -x^2\cos x + 2\int x\cos x\,dx$$

$$= -x^2\cos x + 2\left[x\int \cos x - \int\left[\cos x\,dx\,\frac{d}{dx}(x)\right]dx\right] + c_1$$

$$= -x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right]$$

$$= -x^2\cos x + 2\,[x\sin x + \cos x] + c$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + c$$

GROUP (Q)-HOME WORK PROBLEMS

1) $\int e^{3x}\cos 5x\,dx$

Ans. $e^{3x}\left(\frac{3\cos 5x + 5\sin 5x}{34}\right) + c$

2) $\int e^{5x}\sin 3x\,dx$

Ans. $e^{5x}\left(\frac{5\sin 3x - 3\cos 3x}{34}\right) + c$

3) $I = \int 3^x \cos 2x\,dx$

Ans. $= \cos 2x\int 3^x\,dx - \int\left[\int 3^x dx\,\frac{d}{dx}(\cos 2x)\right]dx$

$$= \cos 2x\,\frac{3^x}{\log 3} - \int\left[\frac{3^x}{\log 3}(-2\sin 2x)\right]dx$$

$$= \frac{3^x\cos 2x}{\log 3} + \frac{2}{\log 3}$$

$$\left[\sin 2x\int 3^x dx - \int 3^x dx\,\frac{d}{dx}(\sin 2x)\,dx\right] + c_1$$

$$= \frac{3^x\cos 2x}{\log 3} + \frac{2}{\log 3}$$

$$\left[\sin 2x\,\frac{3^x}{\log x} - \int \frac{3^x}{\log 3}\cdot 2\cos 2x\,dx\right]$$

$$I = \frac{3^x\cos 2x}{\log 3} + \frac{2\sin 2x\,3^x}{(\log 3)^2} - \frac{4}{(\log 3)^2}\,I + c$$

$\therefore$ $I + \frac{4}{(\log 3)^2}\,I = \frac{3^x\cos 2x\log 3 + 2\sin 2x\,3^x}{(\log 3)^2}$

$\therefore$ $I\left(\frac{(\log 3)^2 + 4}{(\log 3)^2}\right)$

$$= 3^x\,\frac{(\log 3\cos 2x + 2\sin 2x)}{(\log 3)^2} + c$$

$$I = \frac{3^x}{4 + (\log 3)^2}\,[\log 3\cdot\cos 2x + 2\sin 2x] + c$$

4) $I = \int 2^x \sin 5x\,dx$

Ans. $= \sin 5x\int 2^x\,dx - \int 2^x\,dx\,\frac{d}{dx}(\sin 5x)\,dx$

$$= \sin 5x\cdot\frac{2^x}{\log 2} - \int \frac{2^x}{\log 2}\,5\cos 5x\,dx + c_1$$

$$= \frac{\sin 5x\cdot 2^x}{\log 2} - \frac{5}{\log 2}\int 2^x\cos 5x\,dx + c_1$$

$$= \frac{2^x\sin 5x}{\log 2} - \frac{5}{\log 2}$$

$$\left[\cos 5x \int 2^x - \int\left[\int 2^x dx \ \frac{d}{dx}(\cos 5x)\ dx\right]\right]$$

$$= 2^x \frac{\sin 5x}{\log 2} - \frac{5}{\log 2} \times$$

$$\left[\cos 5x \ \frac{2^x}{\log 2} - \int \frac{2^x}{\log 2}(-5\ \sin 5x)\ dx\right]$$

$$I = \frac{2^x}{\log 2}\ \sin 5x - \frac{5 \cdot 2^x\ \cos 5x}{(\log 2)^2} - \frac{25}{(\log 2)^2}\ I + c$$

$$\therefore \quad I + \frac{25}{(\log 2)^2} I = \frac{2^x \sin 5x \log 2 - 5.2^x \cos 5x}{(\log 2)^2} + c$$

$$\therefore \quad I\left(\frac{(\log 2)^2 + 25}{(\log 2)^2}\right) = \frac{2^x(\log 2 \sin 5x - 5\cos 5x)}{(\log 2)^2} + c$$

$$\therefore \ I = \frac{2^x}{[25 + (\log 2)^2]}\ (\log 2 \cdot \sin 5x - 5\ \cos 5x) + c$$

5) $I = \int e^{3x}\ \cos(bx + c)\ dx$

Ans. $= \cos(bx + c) \int e^{3x}\ dx -$

$$\int\left[\int e^{3x}\ dx \frac{d}{dx}\left[\cos(bx + c)\right]\right]$$

$$= \cos(bx + c)\ \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\ (-\sin(bx + c)) \cdot b \cdot dx$$

$$= \frac{e^{3x}}{3}\ \cos(bx + c) + b \int \frac{e^{3x}}{3} \sin(bx + c)\ dx + c_1$$

$$= \frac{e^{3x}}{3}\ \cos(bx + c) + \frac{b}{3}$$

$$\left\{\begin{matrix}\left[\sin(bx + c)\right]\int {}^{3x}dx - \\ \int\left[\int e^{3x}dx \frac{d}{dx}\sin(bx + c)\,dx\right]\end{matrix}\right\}$$

$$= \frac{e^{3x}}{3}\ \cos(bx + c) + \frac{b}{3}\ \sin(bx + c)\ \left(\frac{e^{3x}}{3}\right) -$$

$$\frac{b}{3}\int \frac{e^{3x}}{3} \cdot b \cos(bx + c)$$

$$I = \frac{e^{3x}}{3}\ \cos(bx + c) + \frac{e^{3x}}{9} \cdot b \sin(bx + c) -$$

$$-\frac{b^2}{9} \cdot I + K$$

$$\therefore \quad \left(\frac{9 + b^2}{9}\right) I = \frac{e^{3x}}{9}\ [3\cos(bx + c) + b \sin$$

$$(bx + c)] + K$$

$$I = \frac{e^{3x}}{9 + b^2}\ [3\cos(bx + c) + b\sin(bx + c)] + K$$

6) $I = \int \operatorname{cosec} x \cdot \operatorname{cosec}^2 x\ dx$

Ans. $= \operatorname{cosec} x \int \operatorname{cosec}^2 x\ dx -$ ‘

$$\int\left[\int \operatorname{cosec}^2 x\ dx\ \frac{d}{dx}\ (\operatorname{cosec} x)\right] dx$$

$$= -\operatorname{cosec} x \cot x + \int \cot x\ \ (-\operatorname{cosec} x \cot x)\ dx$$

$$= -\operatorname{cosec} x \cot x - \int (\operatorname{cosec}^2 x - 1)\ \operatorname{cosec} x\ dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x\ + \log\left|\tan \frac{x}{2}\right|$$

$$I = -\operatorname{cosec} x \cot x + \log\left|\tan \frac{x}{2}\right| - I$$

$$= \frac{1}{2}\ \left(-\operatorname{cosec} x \cot x + \log\left|\tan\ \frac{x}{2}\right|\right) + c$$

7) $\int \cos(2\log x)\ dx$

Ans. Put $2\log x = t \quad \therefore\ \log x^2 = t \ \ \therefore\ e^{t/2} = x$

$$\frac{2}{x}\ dx = dt$$

$$dx = \frac{dt}{2} \cdot e^{t/2}$$

$$I = \frac{1}{2}\ \int \cos t \cdot e^{t/2} dt$$

$$I = \frac{1}{2}\ \left[\cos t \int e^{t/2}\ dt - \int\int e^{t/2} dt\ \frac{d}{dt}\ (\cos t)\ dt\right]$$

$$= \frac{1}{2}\ \left[2\cos t \cdot e^{t/2} - \int 2e^{t/2} \cdot (-\sin t)\right] dt + c_1$$

$$= \cos t\ e^{t/2} +$$

$$\left[\sin t \int e^{t/2} dt - \int e^{t/2} dt\ \frac{d}{dt}(\sin t)\right]$$

$$= \cos t \cdot e^{t/2} + 2 \sin t\ e^{t/2} - 2\int e^{t/2} \cos t\ dt + c$$

$$I = \cos t \cdot e^{t/2} + 2 \sin t\ e^{t/2} - 4I + c$$

$$I = \frac{x}{5}\ [\cos(2\ \log x) + 2\ \sin\ (2\ \log x)] + c$$

8) $\int e^x \sin x \cos x\ dx$

Ans. $I = \frac{1}{2}\ \int e^x \sin 2x\ dx$

$$= \frac{1}{2}\ \left\{\sin 2x \int e^x dx - \left[\int e^x dx \frac{d}{dx}(\sin 2x)\right] dx\right\}$$

$$= \frac{1}{2}\ \left\{\sin 2x\, e^x \int 2e^x \cos 2x \cdot dx\ \right\}$$

$$= \frac{1}{2}\begin{Bmatrix}\sin 2x\, e^x \\ -2\left[\cos 2x\ \int e^x dx - \int e^x dx\ \frac{d}{dx}(\cos 2x)\right]\end{Bmatrix}$$

$$= \frac{1}{2}$$

$$\left\{\sin 2x\ ex\ \ -2\ \cos 2x\ ex + \int -4\ \sin 2x\ e^x\right\} + c$$

$$= \frac{1}{2}$$

$$\left\{\sin 2x\, e^x - 2\ \cos 2x \cdot ex\ -\ 4\int e^x \sin 2x\ dx\right\} + c$$

$$= \frac{1}{2}\left\{\sin 2x\, e^x - 2\cos 2x\, e^x - 8\int e^x \sin x \cos x\, dx\right\}$$

$$= \frac{e^x}{2}\{\sin 2x - 2\cos 2x\} - 4I$$

$\therefore \quad 5I = \frac{e^x}{2}[\sin 2x - 2\cos 2x]$

$\therefore \quad I = \frac{e^x}{10}[\sin 2x - 2\cos 2x]$

GROUP (R)-HOME WORK PROBLEMS

1) $\int e^x (\sin x + \cos x)\, dx$

Ans. $= e^x \sin x + c$

2) $\int e^x \frac{\cos x + \sin x}{\cos^2 x}\, dx$

Ans. $= \int e^x (\sec x + \tan x)$

$= e^x \sec x + c$

3) $\int e^x \frac{2 + \sin 2x}{1 + \cos 2x}\, dx$

Ans. $= \int e^x \frac{2 + 2\sin x \cos x}{2\cos^2 x}$

$= \int e^x \left(\sec^2 x + \tan x\right)$

$= e^x \tan x + c$

4) $\int e^x \frac{2 - \sin 2x}{1 - \cos 2x}\, dx$

Ans. $= \int e^x \frac{2 - 2\sin x \cos 2x}{2\sin^2 x}\, dx$

$= \int e^x \left(\cos ec^2 x - \cot x\right) dx$

$= -e^x \cot x + c$

5) $\int e^x \left(\frac{1 + \sin 2x}{1 + \cos 2x}\right) dx$

Ans. put 2x = t ∴ 2 dx = dt

Let, I $= \int e^{2x} \frac{1 + \sin 2x}{1 + \cos 2x}\, dx$

$= \int e^t \frac{1 + \sin t}{1 + \cos t} \times \frac{dt}{2}$

$= \frac{1}{2} \int e^t \left(\frac{1}{2\cos^2 t/2} + \frac{2\sin t/2 \cot t/2}{2\cos^2 t/2}\right) dt$

$= \frac{1}{2} \int e^t \left(\frac{1}{2} \sec^2 t/2 + \tan t/2\right) dt$

$= \frac{1}{2} e^t \tan \frac{1}{2} + c$

$= \frac{1}{2} e^{2x} \tan x + c$

6) $\int e^x \frac{(x - 1)^2}{(x^2 + 1)^2}\, dx$

Ans. $= \int e^x \left(\frac{x^2 - 2x + 1}{x^4 + 2x^2 + 1}\right) dx$

$= \int e^x \left(\frac{x^2 + 1}{(x^2 + 1)^2} - \frac{2x}{(x^2 + 1)^2}\right) dx$

$= \int e^x \left(\frac{1}{x^2 + 1} - \frac{2x}{(x^2 + 1)^2}\right) dx$

$= \frac{e^x}{x^2 + 1} + c$

7) $\int \frac{xe^x}{(x + 1)^2}\, dx$

Ans. $= \int e^x \left(\frac{x + 1}{(x + 1)^2} - \frac{1}{(x + 1)^2}\right) dx$

$= \int e^x \left(\frac{1}{x + 1} - \frac{1}{(x + 1)^2}\right) dx$

$= \frac{e^x}{x + 1} + c$

8) $\int \left[\sin(\log x) + \cos(\log x)\right] dx$

Ans. Let I = $\int \left[\sin(\log x) + \cos(\log x)\right] dx$

put log x = t. Then x = e^t ∴ dx = e^t dt

∴ I $= \int (\sin t + \cos t) e^t dt$

If f (t) = sin t, the f'(t) = cos t

$= \int e^t \left[f(t) + f'(t)\right]$

$= e^t f(t) + c = e^t \sin t + c$

= x . sin (log x) + c

9) $\int e^x \left(\frac{1}{x} + \log x\right) dx$

Ans. put f(x) = log x

∴ f' (x) $= \frac{1}{x}$

∴ I = $\int e^x \left[f(x) + f'(x)\right] dx$

$= e^x f(x) + c$

$= e^x \log x + c$

10) $\int e^{\sin^{-1}x}\left[\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]dx$

Ans. put $\sin^{-1} x = t$.Then $x = \sin t$ and

$$\frac{dx}{\sqrt{1-x^2}} = dt$$

$$\therefore \quad I = \int e^{\sin{-1}x}\left(x+\sqrt{1-x^2}\right).\frac{dx}{\sqrt{1-x^2}}$$

$$= \int e^t\left(\sin t+\sqrt{1-\sin^2 t}\right).dt$$

$$= \int e^t(\sin t+\cos t)\,dt$$

let f(t) = sin t. Then f'(t) = cos t

$$\therefore \quad I = \int e^t\left[f(t)+f'(t)\right]dt$$

$$= e^t . f(t) + c = e^t . \sin t + c$$

$$= e^{\sin^{-1}x}.x + c = x.e^{\sin^{-1}x} + c$$

11) $\int e^x\ (\tan x - \log \cos x)\ dx$

Ans. $f(x) = -\log(\cos x)$

$$\therefore \quad f'(x) = \frac{-1}{\cos x}\cdot(-\sin x) = \tan x$$

$$= -e^x(\log \cos x) + C$$

12) $\int e^x(\cot x+\log\sin x)\,dx$

Ans. Let $I = \int e^x(\cot x+\log\sin x)\,dx$

Put f(x) = log sin x.Then

$$f'(x) = \frac{d}{dx}(\log\sin x) = \frac{1}{\sin x}.\cos x = \cot x$$

$$\therefore \quad I = \int e^x\left[f'(x)+f(x)\right]dx$$

$$= e^x . f(x) + c = e^x . \log\sin x + c$$

GROUP (S)-HOME WORK PROBLEMS

1) $\int\sqrt{x^2-6x+10}\ dx$

Ans. $= \int\sqrt{(x-3)^2+1^2}\ dx$

$$= \frac{x-3}{2}\sqrt{x^2-6x+10} + \frac{1}{2}\log|x-3+$$

$$\sqrt{x^2-6x+10}\ | + c.$$

2) $\int\sqrt{9x^2+6x+7}\ dx$

Ans. $= 3\int\sqrt{x^2+\frac{2x}{3}+\frac{7}{9}}\ dx$

$$= 3\int\sqrt{\left(x+\frac{1}{3}\right)^2+\left(\sqrt{\frac{2}{3}}\right)^2}\ dx$$

$$= 3\left[\frac{x+\frac{1}{3}}{2}\sqrt{x^2+\frac{2}{3}x+\frac{7}{9}}+\frac{2}{3\times 2}\log\left|x+\frac{1}{3}+\sqrt{x^2+\frac{2}{3}x+\frac{7}{9}}\right|\right]$$

$$= \frac{3x+1}{2}\sqrt{x^2+\frac{2}{3}x+\frac{7}{9}}+\log$$

$$\left|x+\frac{1}{3}+\sqrt{x^2+\frac{2}{3}x+\frac{7}{9}}\right|+c$$

3) $\int\sqrt{5-4x-x^2}dx$

Ans. $= \int\sqrt{-(x^2+4x+4-9)}\ dx$

$$= \int\sqrt{3^2-(x+2)^2}\ dx$$

$$= \frac{x+2}{2}\sqrt{5-4x-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x+2}{3}\right)+c$$

4) $\int\sqrt{4+3x-2x^2}\ dx$

Ans. $= \sqrt{2}\int\sqrt{-\left(x^2-\frac{3x}{2}-2\right)}\ dx$

$$= \sqrt{2}\int\sqrt{-\left(x^2-\frac{3x}{2}+\frac{9}{16}-\frac{41}{16}\right)}\ dx$$

$$= \sqrt{2}\int\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2-\left(x-\frac{3}{4}\right)^2}\ dx$$

$$= \sqrt{2}\left[\frac{x-\frac{3}{4}}{2}\sqrt{\frac{3x}{2}+2-x^2}+\frac{41}{16\times 2}\sin^{-1}\left(\frac{x-\frac{3}{4}}{\frac{\sqrt{41}}{4}}\right)\right]+c$$

$$= \frac{4x-3}{8}\sqrt{4+3x-2x^2} + \frac{41}{16\sqrt{2}}\sin^{-1}\left(\frac{4x-3}{\sqrt{41}}\right) + c$$

5) $\int x\sqrt{x^4+a^4}\,dx$

Ans. Let, $I = \int x\sqrt{x^4+a^4}\,dx$

$x^2 = t$

∴ $2x\,dx = dt$

∴ $x\,dx = \frac{dt}{2}$

So, $I = \int x\sqrt{x^4+a^4}\,\frac{dt}{2}$

$$= \frac{1}{2}\int\sqrt{t^2+\left(a^2\right)^2}\,dt$$

$$= \frac{1}{2}\left[\frac{t}{2}\sqrt{t^2+a^4} + \frac{\left(a^2\right)}{2}\log\left|t+\sqrt{t^2+a^4}\right|\right]$$

$$= \frac{1}{2}\left[\frac{x^2}{2}\sqrt{x^2+a^4} + \frac{a^4}{2}\log\left|x^2+\sqrt{x^2+a^4}\right|\right] + c$$

6) $\int x\sqrt{x^4-x^2+1}\,dx$

Ans. put $x^2 = t$

$2xdx = dt$

$$I = \frac{1}{2}\int\sqrt{t^2-t+1}\,dt$$

$$= \frac{1}{2}\int\sqrt{\left(t-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}\,dt$$

$$= \frac{1}{2}\times$$

$$\left[\frac{t-\frac{1}{2}}{2}\sqrt{t^2-t+1} - \frac{3}{4\times 2}\log\left|t-\frac{1}{2}+\sqrt{t^2-t+1}\right|\right] + c$$

$$= \frac{2x^2-1}{8}\sqrt{x^4-x^2+1} - \frac{3}{16}\log\left|x^2-\frac{1}{2}+\sqrt{x^4-x^2+1}\right| + c$$

7) $\int e^x\sqrt{5e^{2x}-4e^x-3}\,dx$

Ans. $e^x = t$

$e^x dx = dt$

$$\int\sqrt{5t^2-4t-3}\,dt$$

$$= \sqrt{5}\int\sqrt{t^2-\frac{4}{5}t+\frac{4}{25}-\frac{4}{25}-\frac{3}{5}}\,dt$$

$$= \sqrt{5}\int\sqrt{\left(t-\frac{2}{5}\right)^2-\left(\frac{\sqrt{19}}{5}\right)^2}$$

$$= \sqrt{5}\left[\frac{t-\frac{2}{5}}{2}\sqrt{t^2-\frac{4}{5}t-\frac{3}{5}} - \frac{19}{25\times 2}\log\left|t-\frac{2}{5}+\sqrt{t^2-\frac{4}{5}t-\frac{3}{5}}\right|\right] + c$$

$$= \sqrt{5}\left(\frac{5e^x-2}{10}\right)\sqrt{e^{2x}-\frac{4}{5}e^x-\frac{3}{5}} - \frac{19\sqrt{5}}{50}\log\left|e^x-\frac{2}{5}+\sqrt{e^{2x}-\frac{4}{5}e^x-\frac{3}{5}}\right| + c$$

8) $\int\sqrt{9-\cos^2 x}\,\sin x\,dx$

Ans. put $\cos x = t$

$-\sin x\,dx = dt$

$$-\int\sqrt{3^2-t^2}\,dt$$

$$= -\left[\frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\left(\frac{t}{3}\right)\right] + c$$

$$= \frac{\cos x}{2}\sqrt{9-\cos^2 x} - \frac{9}{2}\sin^{-1}\left(\frac{\cos x}{3}\right) + c$$

9) $\int\sin x\sqrt{\cos^2 x-2\cos x+2}\,dx$

Ans. put $\cos x = t$

$-\sin x\,dx = dt$

$$-\int\sqrt{t^2-2t+2}\,dt$$

$$= -\int\sqrt{(t-1)^2+1^2}\,dt$$

$$= \left[\frac{t-1}{2}\sqrt{t^2-2t+2} + \frac{1}{2}\log\left|t-1+\sqrt{t^2-2t+2}\right|\right]$$

$$= \left(\frac{\cos x - 1}{2}\right)\sqrt{\cos^2 x - 2\cos x + 2} - \frac{1}{2}$$

$$\log|\cos x - 1 + \sqrt{\cos^2 x - 2\cos x + 2}\,|$$

10) $\int \sqrt{a^2 - \sin^2 x}\,\cos x\,dx$

Ans. put sinx = t
cosx dx = dt

$$\int\sqrt{a^2 - t^2}\,dt$$

$$= \frac{t}{2}\sqrt{a^2 - t^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{t}{a}\right) + c$$

$$= \frac{\sin x}{2}\sqrt{a^2 - \sin^2 x} + \frac{a^2}{2}\sin^{-1}\left(\frac{\sin x}{a}\right) + c$$

11) $\int \cos x\sqrt{\cos 2x}\,dx$

Ans. $= \int \cos x\sqrt{1 - 2\sin^2 x}\,dx$

sinx = t
cosx dx = dt

$$= \int\sqrt{1^2 - (\sqrt{2}t)^2}\,dt$$

$$= \frac{\sqrt{2}t}{2\sqrt{2}}\sqrt{1 - (\sqrt{2}t)^2} + \frac{1}{2\sqrt{2}}\sin^{-1}\left(\frac{\sqrt{2}t}{1}\right) + c$$

$$= \frac{\sin x}{2}\sqrt{\cos 2x} + \frac{1}{2\sqrt{2}}\sin^{-1}\left(\sqrt{2}\sin x\right) + c$$

GROUP (T)-HOME WORK PROBLEMS

1) $\int \cos^8 x\,dx$

Ans. $= \frac{\cos^7 x\sin x}{8} + \frac{7}{8}\int \cos^6 x\,dx + c$

$$= \frac{\sin x\cos^7 x}{8} + \frac{7}{8}\int (1 - \sin^2 x)^3 dx + c$$

$$= \frac{\sin x\cos^7 x}{8} + \frac{7}{8}$$

$$\int (1 - 3\sin^2 x + 3\sin^4 x + \sin^6 x)\,dx + c$$

$$= \frac{\sin x\cos^7 x}{8} + \frac{7}{8}\int 1 - 3\left(\frac{1 - \cos 2x}{2}\right) +$$

$$3\left(\frac{1 - \cos 2x}{2}\right)^2 + \left(\frac{1 - \cos 2x}{2}\right)^3 dx + c$$

$$= \frac{\sin x\cos^7 x}{8} + \frac{7}{8}x - \frac{21}{16}x + \frac{7}{32}\sin 2x + \frac{7}{8}\times$$

$$\frac{3}{4}\int (1 - 2\cos 2x + \cos^2 2x)dx$$

$$+ \frac{1}{8}\times\frac{7}{8}\int 1 - 3\cos 2x + 3\cos^2 2x + \cos^3 2x$$

$+ c$

$$= \frac{\sin x\cos^7 x}{8} - \frac{7}{16}x + \frac{7}{16}\sin x\cos x +$$

$$\frac{21}{32}\left(x - \frac{2\sin 2x}{2} + \frac{x - \frac{\sin 4x}{4}}{2}\right) + \frac{7}{64}$$

$$\left(x - \frac{3\sin 2x}{2} + \frac{3\left(x + \frac{\sin 4x}{4}\right)}{2} + \frac{3\sin 2x}{8} - \frac{\sin 4x}{16}\right)$$

$$= \frac{\sin x\cos^7 x}{8} - \frac{169x}{128} + \frac{7}{16}\sin x\cos x - \frac{21}{32}$$

$$\sin x\cos x - \frac{21}{32}\times\frac{\sin 2x\cos 2x}{8} - \frac{21}{128}$$

$$(2\sin x\cos x) + \frac{7}{64}\times\frac{3}{8}(2\sin x\cos 2x)$$

$$+ \frac{7}{64}\times\frac{3}{4}(2\sin x\cos x) - \frac{7}{64}\times\frac{1}{8}$$

$$(2\sin 2x\cos 2x)$$

$$= \frac{\sin x\cos^7 x}{8} - \frac{189}{128}x - \frac{14}{16}\sin x\cos x - \frac{21}{64}$$

$$\sin x\cos x + \frac{21}{256}\sin x\cos x - \frac{41}{128}$$

$$+ \frac{21}{256} - \frac{7}{512}(\sin 2x\cos 2x)$$

$= \sin x$

$$\left(\frac{\cos^7 x}{8} - \frac{7\cos^5 x}{48} + \frac{35\cos^3 x}{192} + \frac{105\cos x}{385}\right)$$

$$+ \frac{105x}{385} + c$$

GROUP (U)-HOME WORK PROBLEMS

1) $\displaystyle\int \frac{5x + 2}{(x - 2)\ (x - 1)}\ dx$

Ans. $5x + 2 = A(x - 1) + B(x - 2)$

$x = 2, \quad x = 1$

$12 = A \quad 7 = -B$

$B = -7$

$\therefore \quad I = \displaystyle\int \frac{12}{x - 2}\ dx\ -7 \int \frac{1}{x - 1}\ dx$

$= 12 \log |x - 2| - 7\log |x - 1| + c$

2) $\displaystyle\int \frac{3x - 2}{x^2 - 3x + 2}\ dx$

Ans. $= \displaystyle\int \frac{3x - 2}{(x - 2)(x - 1)} dx$

$3x - 2 = A(x - 1) + B(x - 2)$

$x = 2, x = 1$

$A = 4 -B = 1$

$B = -1$

$= \displaystyle\int \frac{4}{x - 2} + \int \frac{-1}{x - 1}$

$= 4 \log |x - 4| - \log |x - 1| + c$

3) $\displaystyle\int \frac{4x}{2x^2 + 2x - x - 1} dx$

Ans. $= 4 \displaystyle\int \frac{x}{2x(x + 1) - 1(x + 1)} dx$

$= 4 \displaystyle\int \frac{x}{(2x - 1)\ (x + 1)} dx$

$4x = A(x + 1) + B(2x - 1)$

put $x = \frac{1}{2}, \qquad x = -1$

$\frac{3}{2} A = 4 \times \frac{1}{2}, \quad -4 = -3\ B$

$A = \frac{4}{3}, \quad B = \frac{4}{3}$

$\therefore \quad I = \frac{4}{3} \displaystyle\int \frac{1}{2x - 1} + \frac{4}{3} \int \frac{1}{x + 1} dx$

$= \frac{4}{3}\ \frac{\log|2x - 1|}{2} + \frac{4}{3} \log |x + 1| + c$

$= \frac{2}{3} \log |2x - 1| + \frac{4}{3} \log |x + 1| + c$

4) $\displaystyle\int \frac{1}{2x^2 - 6x - x + 3}\ dx$

Ans. $= \displaystyle\int \frac{1}{2x(x - 3) -1\ (x - 3)}\ dx$

$= \displaystyle\int \frac{1}{(2x - 1)\ (x - 3)}\ dx$

$1 = A\ (x - 3) + B(2x - 1)$

put $x = \frac{1}{2}, \qquad x = 3$

$1 = A\left(\frac{1}{2} - 3\right), \quad 1 = 5B$

$A = \frac{-2}{5}, B = \frac{1}{5}$

$\therefore \quad I = \frac{1}{5} \displaystyle\int \frac{1}{x - 3} + \frac{-2}{5} \int \frac{1}{2x - 1}\ dx$

$= \frac{1}{5} \log \left|\frac{x - 3}{(2x - 1)}\right| + c$

5) $\displaystyle\int \frac{1}{x(x - 2)\ (x - 3)}\ dx$

Ans. Let, $\dfrac{1}{x(x - 2)(x - 3)} = \dfrac{A}{x} + \dfrac{B}{x - 2} + \dfrac{C}{x - 3}$

$A\ (x - 2)(x - 3) + B\ x(x - 3) + C(x - 2)x = 1$

Put $x = 0$, put $x = 2$, put $x = 3$

$A \cdot 6 = 1 Bx - 2 = 1 \qquad C \cdot 3 = 1$

$A = \frac{1}{6} \quad B = \frac{-1}{2}\ C = \frac{1}{3}$

$\therefore\ I = \frac{1}{6} \displaystyle\int \frac{1}{x} + \frac{-1}{2} \int \frac{1}{x - 2} + \frac{1}{3} \int \frac{1}{x - 3}$

$= \frac{1}{6} \log |x| + \frac{1}{3} \log |x - 3| - \frac{1}{2} \log |x - 2| + c.$

6) $\displaystyle\int \frac{x}{(x + 2)\ (x + 2)\ (x + 3)}\ dx$

Ans. Let, $I = \displaystyle\int \frac{x}{(x + 2)\ (x + 2)\ (x + 3)}\ dx$

$= \dfrac{A}{x + 1} + \dfrac{B}{x + 2} + \dfrac{C}{x + 3}$

$\therefore \quad x = A(x + 2)\ (x + 3) + B(x + 1)(x + 3) + C(x + 1)\ (x + 2)$

Put $x = -1$, put $x = -2$, put $x = -3$

$A = \frac{-1}{2}, B = 2, \quad C = \frac{-3}{2}$

∴ $I= 2\log|x + 2| - \frac{1}{2}\log|x + 1| - \frac{3}{2}\log$
$|x + 3| + c$

7) $\int \frac{x + 1}{x(x - 2)(x + 3)}dx$

Ans. Let, $\frac{x + 1}{x(x - 2)(x + 3)} = \frac{A}{x} + \frac{B}{(x - 2)} + \frac{C}{(x + 3)}$
$x + 1 = A(x - 2)(x + 3) + Bx(x + 3) + Cx(x - 2)$
Put $x = 0$, Put $x = 2$, put $x = -3$

$A = \frac{-1}{6}$ $3 = 10B, -2 = 15C$

$B = \frac{3}{10}$ $C = \frac{-2}{15}$

∴ $I = \frac{3}{10}\log|x - 2| - \frac{1}{6}\log|x| - \frac{2}{15}\log$
$|x + 3| + c$

8) $\int \frac{x^2 + 2}{(x + 2)(x + 1)(x + 3)}dx$

Ans. $\frac{x^2 + 2}{(x + 2)(x + 1)(x + 3)} = \frac{A}{(x + 2)} + \frac{B}{(x + 1)} +$

$\frac{C}{(x + 3)}$

∴ $x^2 + 2 = A(x + 1)(x + 3) + B(x + 2)(x + 3) +$
$C(x + 1)$
$(x + 2)$
Put : $x = -2$, $x = -1$ $x = -3$
$-A = 6$, $2B = 3, 2C = 11$

∴ $A = -6$, $B = \frac{3}{2}, C = \frac{11}{2}$

∴ $I = \frac{3}{2}\log|x + 1| + \frac{11}{2}\log|x + 3| - 6\log$
$|x + 2| + c$

9) $\int \frac{5x^2 - 1}{x(x - 1)(x + 1)}dx$

Ans. Let, $\int \frac{5x^2 - 1}{x(x - 1)(x + 1)}dx$

$= \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x + 1}$
$A(x - 1)(x + 1) + B(x + 1)x + C(x - 1)x$
$= 5x^2 - 1$
Put $x = 0$, $x = 1$ $x = -1$
$-A = -1$ $2B = 4$ $2C = 4$
$A = 1$ $B = 2$ $C = 2$

∴ $I = \log|x| + 2\log|x - 1| + 2\log|x + 1| + C$

Put $x^2 = t$

10) $\int \frac{x^2 + 37}{(x^2 - 7)(x^2 + 4)}dx$

Ans. Let, $\int \frac{x^2 + 37}{(x^2 - 7)(x^2 + 4)}dx$
put $x^2 = t$
So,

$\frac{t + 37}{(t - 7)(t + 4)} = \frac{A}{(t - 7)} + \frac{B}{(t + 4)}$
$A(t + 4) + B(t - 7) = t + 37$
Put $t = 7$, $t = -4$
$11A = 44$ $-11B = 33$
$A = 45$ $B = -3$

∴ $I = \int\left(\frac{4}{t - 7}\frac{-3}{t + 4}\right)dx$

$= 4\int \frac{1}{x^2 - 7}dx - 3\int \frac{1}{x^2 + 4}dx$

$= \frac{2}{\sqrt{7}}\log\left|\frac{x - \sqrt{7}}{x + \sqrt{7}}\right| - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

11) $\int \frac{2x^2 - 1}{(x^2 + 4)(x^2 + 5)}dx$

Ans. put $x^2 = t$
So,

$\frac{2t^2 - 1}{(t + 4)(t + 5)} = \frac{A}{t + 4} + \frac{B}{t + 5}$
Put $x^2 = t$
$A(t + 5) + B(t + 4) = 2t - 1$
put $t = -4$ $t = -5$
$A = -9$ $B = 11$

∴ $I = 11\int \frac{1}{x^2 + 5}dx - 9\int \frac{1}{x^2 + 4}dx$

$= \frac{11}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) - \frac{9}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

12) $\int \frac{\sec^2 x}{(2 + \tan x)(3 + \tan x)}dx$

Ans. put $\tan x = t$
$\sec^2 x\,dx = dt$

∴ $I = \int \frac{dt}{(2 + t)(3 + t)}$

Let, $\dfrac{A}{2+t} + \dfrac{B}{3+t} = \dfrac{1}{(2+t)(3+t)}$

$\therefore$ $A(3+t) + B(2+t) = 1$

put $t = -2$, $t = -3$

$A = 1$ $B = -1$

$I = \log|2+\tan x| - \log|3+\tan x| + c$

$= \log\left|\dfrac{2+\tan x}{3+\tan x}\right| + c$

13) put $\log x = t$

Ans. $\dfrac{1}{x}\,dx = dt$

$\displaystyle\int \frac{t}{(2+t)(3+t)}\,dt$

$\dfrac{A}{2+t} + \dfrac{B}{3+t} = \dfrac{t}{(2+t)(3+t)}$

$A(3+t) + B(2+t) = t$

put $t = -2$, $t = -3$

$A = -2$ $B = 3$

$= 3\log|3+\log x| - 2\log|2+\log x| + c$

$= \log\left|\dfrac{(3+\log x)^3}{(2+\log x)^2}\right| + c$

14) $\sin x = t$

Ans. $\displaystyle\int \frac{t^2}{t^2 - 5t + 6}\,dx$

$= \displaystyle\int \frac{t^2}{(t-2)(t-3)}\,dx$

$A(t-3) + B(t-2) = t^2$

put, $t = 2$, $t = 3$

$\therefore$ $-A = 4$ $B = 9$

$\therefore$ $A = -4$

$I = 9\log|\sin x - 3| - 4\log|\sin x - 2| + c$

15) $\sin x = t$

Ans. $\cos x\,dx = dt$

$\therefore$ $I = \displaystyle\int \frac{dt}{(1+t)(2+t)(3+t)}$

Let, $\dfrac{1}{(1+t)(2+t)(3+t)}$

$= \dfrac{A}{1+t} + \dfrac{B}{2+t} + \dfrac{C}{3+t}$

$A(2+t)(3+t) + B(t+1)(3+t) + C(t+1)(t+2) = 1$

$t = -1$, $t = -2$ $t = -3$

$2A = 1$ $-B = 1$ $C = \dfrac{1}{2}$

$A = \dfrac{1}{2}$ $B = -1$

$\therefore$ $I = \dfrac{1}{2}\log|1+\sin x| - \log|2+\sin x| + \dfrac{1}{2}\log$ $|3+\sin x| + c$

16) $\displaystyle\int \frac{\sin 2x}{\cos^2 x + 4\cos x + 3}\,dx$

Ans. $= 2\displaystyle\int \frac{\sin x\cos x}{\cos^2 x + 4\cos x + 3}\,dx$

$\cos x = t$

$-\sin x\,dx = dt$

$= -2\displaystyle\int \frac{t}{t^2 + 4t + 3}\,dt$

$= -2\displaystyle\int \frac{t}{(t+3)(t+1)}\,dt$

$A(t+1) + B(t+3) = -2t$

$t = -3$ $t = -1$

$\therefore$ $6 = -2A$ $B = 1$

$\therefore$ $A = -3$

$\therefore$ $I = \log|\cos x + 1| - 3\log|\cos x + 3| + c$

17) $\displaystyle\int \frac{\sec^2 x}{\tan^2 x - 3\tan x + 2}\,dx$

Ans. $\tan x = t$

$\sec^2 x\,dx = dt$

$\therefore$ $I = \displaystyle\int \frac{dt}{(t-2)(t-1)}$

Let, $\dfrac{A}{(t-2)} + \dfrac{B}{(t-1)} = \dfrac{1}{(t-2)(t-1)}$

$A(t-1) + B(t-2)\,] = 1$

$t = 2$, $t = 1$

$A = 1$, $B = -1$

$\therefore$ $I = \log|\tan x - 2| - \log|\tan x - 1| + c$

$= \log\left|\dfrac{\tan x - 2}{\tan x - 1}\right| + c$

18) $\displaystyle\int \frac{1+\log x}{x(2+\log x)(3+2\log x)}$

Ans. Let, $I = \displaystyle\int \frac{1+\log x}{x(2+\log x)(3+2\log x)}$

put $\log x = t$

$\therefore$ $\dfrac{1}{x}\,dx = dt$

$\therefore$ $I = \displaystyle\int \frac{(1+t)}{(2+t)(3+2t)}\,dt$

Let $\frac{(1+t)}{(2+t)(3+2t)} = \frac{A}{(2+t)} + \frac{B}{(3+t)}$

∴ 1 + t = A (3 + 2t) + B (2 + t)

put t = – 2

∴ A = 1

put t = – 3/2

∴ $1 - \frac{3}{2} = B\left(2 - \frac{3}{2}\right)$

∴ $-\frac{1}{2} = B\left(\frac{1}{2}\right)$

∴ B = – 1

So,

$$I = \int\left(\frac{1}{2+t} - \frac{1}{3+2t}\right)dt$$

$$= \log|2+t| - \log\frac{|3+2t|}{2} + c$$

$$= \log|2+\log x| - \frac{1}{2}\log|3+2\log x| + c$$

19) $\int \frac{\log x}{x(1+\log x)(2+\log x)}dx$

Ans. Let, I = $\int \frac{\log x}{x(1+\log x)(2+\log x)}dx$

put log x = t

∴ $\frac{1}{x}dx = dt$

∴ $I = \int \frac{t}{(1+t)(2+t)}\ dt$

Let $\frac{t}{(1+t)(2+t)} = \frac{A}{(1+t)} + \frac{B}{(2+t)}$

∴ t = A (2 + t) + B (1 + t)

put t = – 1

∴ A = – 1

put t = – 2

∴ B = 2

So, $I = \int\left(\frac{-1}{(1+t)} + \frac{2}{(2+t)}\right)\ dt$

$$= -\log|1+t| + 2\log|2+t| + c$$

$$= -\log|1+\log x| + 2\log|2+\log x| + c$$

20) $\int \frac{\sec^2 x}{\left(1\text{-}\tan^2 x\right)(2+\tan x)}dx$

Ans. put tan x = t

$\sec^2 dx = dt$

$$\int \frac{dt}{\left(1\text{-}t^2\right)(2+t)}$$

$$\int \frac{dt}{(1-t)(1+t)(2+t)}$$

$$\frac{1}{(1-t)(1+t)(2+t)} = \frac{A}{(1-t)} + \frac{B}{(2+t)} + \frac{C}{(1+t)}$$

$1 = A(2+t)(1+t) + B(1-t^2) + C(1-t)(2+t)$

put 2 + t = 0

t = – 2

1 = 0 + B (–3)

$B = \frac{-1}{3}$

put $(1 - t^2) = 0$

t = 1

1 = 6A + 0 + 0

$A = \frac{1}{6}$

put 1 + t = 0

t = – 1

1 = 2C

$C = \frac{1}{2}$

$$= \frac{1}{6(1-t)} - \frac{1}{3(2+t)} + \frac{1}{2(1+t)}$$

$$= \int \frac{1}{6(1-t)} - \frac{1}{3(2+t)} + \frac{1}{2(1+t)}dt$$

By substiting

$$\frac{1}{6}\log|(1-t)| - \frac{1}{3}\log|2+t| + \frac{1}{2}\log|1+\tan x| + c$$

$$\frac{1}{6}\log|(1-\tan x)| - \frac{1}{3}\log|2+\tan x|$$

$$+\frac{1}{2}\log|1+\tan x| + c$$

21) $\int \frac{x^2}{x^4 + 10x^2 + 21}\ dx$

Ans. put $x^2 = t$

Let, $\frac{t}{(t+7)(t+3)} = \frac{A}{t+7} + \frac{B}{t+3}$

A(t + 3) + B(t + 7) = t

t = –7, t = –3

$-4A = -7 \qquad 4B = -3$

$A = \frac{7}{4} \quad B = \frac{-3}{4}$

$\therefore \quad I = \frac{7}{4}\int \frac{1}{(x^2+7)} + \frac{-3}{4}\int \frac{1}{(x^2+3)}dx$

$= \frac{\sqrt{7}}{4}\tan^{-1}\left(\frac{x}{\sqrt{7}}\right) - \frac{\sqrt{3}}{4}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

22) $\int \frac{1}{(s^2+a^2)(s^2+b^2)}dx$

Ans. put $s^2 = t$

Let, $\frac{1}{(t+a^2)(t+b^2)} = \frac{A}{(t+a^2)} + \frac{B}{(t+b^2)}$

$A(t+b^2) + B(t+a^2) = 1$

$t = -a^2 \quad t = -b^2$

$A = \frac{1}{b^2-a^2} \qquad B = \frac{1}{a^2-b^2}$

$= \frac{1}{b^2-a^2}\left[\int\frac{1}{s^2+a^2} - \int\frac{1}{s^2+b^2}\right]ds$

$= \frac{1}{b^2-a^2} \times$

$\left[\frac{1}{a}\tan^{-1}\left(\frac{s}{a}\right) - \frac{1}{b}\tan^{-1}\left(\frac{s}{b}\right)\right] + c$

23) $\int \frac{x^2+1}{(x^2+2)(2x^2+1)}dx$

Ans. Let, $I = \int \frac{x^2+1}{(x^2+2)(2x^2+1)}dx$

put $x^2 = t$

Now, let $\frac{t+1}{(t+2)(2t+1)} = \frac{A}{(t+2)} + \frac{B}{(2t+1)}$

$\therefore$ $t + 1 = A(2t+1) + B(t+2)$

put $t = -2$

$\therefore$ $A = 1/3$

$\therefore$ put $t = -\frac{1}{2}$

$\therefore$ $-\frac{1}{2} + 1 = B\left(\frac{-1}{2} + 2\right)$

$\therefore$ $\frac{1}{2} = B(3/2)$

$\therefore$ $B = \frac{1}{3}$

So,

$I = \int\left(\frac{1}{3(t+2)} + \frac{1}{3(2t+1)}\right)dx$

$= \frac{1}{3}\left[\int\frac{1}{\left(x^2+\left(\sqrt{2}\right)^2\right)}dx + \int\frac{1\ dx}{\left(\sqrt{2}x\right)^2+(1)^2}\right] + c$

$= \frac{1}{3\sqrt{2}}\left[\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \tan^{-1}\left(\sqrt{2}\,x\right)\right] + c$

24) $\int \frac{2x^2-1}{(x^2-7)(x^2-5)}dx$

Ans. put $x^2 = t$

Let, $\frac{2t-1}{(t-7)(t-5)} = \frac{A}{(t-7)} + \frac{B}{(t-5)}$ ... (i)

$\therefore$ $2t - 1 = A(t-5) + B(t-7)$

put $t = 7$

$\therefore$ $13 = A \times 2$

$\therefore$ $A = 13/2$

put $t = 5$

$\therefore$ $9 = B(-2)$

$\therefore$ $B = -\frac{9}{2}$

So,

$I = \int\left[\frac{13}{2\left(x^2-7\right)} - \frac{9}{2\left(x^2-5\right)}\right]dx$ [from (i)]

$= \frac{13}{2}\int\frac{1dx}{(x)^2-\left(\sqrt{7}\right)^2} - \frac{9}{2}\int\frac{1}{(x)^2-\left(\sqrt{5}\right)^2}dx$

$= \frac{13}{2}\times\frac{1}{2\sqrt{7}}\log\left|\frac{x-\sqrt{7}}{x+\sqrt{7}}\right| - \frac{9}{2}\times\frac{1}{2\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|$

$+ c$

$= \frac{13}{4\sqrt{7}}\log\left|\frac{x-\sqrt{7}}{x+\sqrt{7}}\right| - \frac{9}{4\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right| + c$