Chapter

10 DEFINITE INTERGRATION

GROUP (A)-HOME WORK PROBLEMS

1) $\int_{-1}^{1} (x^2 + 1)(2x + 3)\, dx$

Ans. $= \int_{-1}^{1} \left[(x^2 + 1) + 1(2x + 3)\right] dx$

$= \int_{-1}^{1} (2x^3 + 3x^2 + 2x + 3)\, dx$

$= 2\int_{-1}^{1} x^3 dx + 3\int_{-1}^{1} x^2 dx + 2\int_{-1}^{1} x\, dx + 3\int_{-1}^{1} 1\, dx$

$= \frac{4}{2}\left[x^4\right]_{-1}^{1} + \frac{3}{3}\left[x^3\right]_{-1}^{1} + \frac{2}{2}\left[x^2\right]_{-1}^{1} + 3[x]_{-1}^{1}\, dx$

$= \frac{1}{2}(1 - 1) + 1(1 + 1) + [1 - 1] + 3[1 + 1]$

$= 2 + 3(2) = 2 + 6 = 8$

2) $\int_0^2 \frac{x^2 - 4}{x^2 + 4}\, dx \quad = \int_0^2 \frac{(x^2 + 4) - 8}{x^2 + 4} dx$

Ans. $= \int_0^2 \frac{x^2 + 4}{x^2 + 4} dx - 8\int_0^2 \frac{1}{x^2 + 2^2}\, dx$

$= \int_0^2 dx - 8\int_0^2 \frac{1}{x^2 + 2^2} dx$

$= [x]_0^2 - \frac{8}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= (2 - 0) - 4\left[\tan^{-1}\left(\frac{2}{2}\right) - 0\right]$

$= 2 - 4\left[\frac{\pi}{4} - 0\right]$

$= 2 - \pi$

3) $\frac{1}{4}\int_{-3/2}^{1/2} \frac{1}{x^2 + 3x + \frac{13}{4}}\, dx$

Ans. $= \frac{1}{4}\int_{-3/2}^{1/2} \frac{dx}{x^2 + 2x.\frac{3}{2} + \frac{9}{4} + \left(\frac{13}{4} - \frac{9}{4}\right)}$

$= \frac{1}{4}\int_{-3/2}^{1/2} \frac{1}{\left(x + \frac{3}{2}\right)^2 + 1^2} dx$

$= \frac{1}{4}\left[\tan^{-1}\left(x + \frac{3}{2}\right)\right]_{-3/2}^{1/2}$

$= \frac{1}{4}[\tan^{-1}(2) - 0]$

$= \frac{1}{4}\tan^{-1}(2)$

4) $\int_1^2 \frac{dx}{(x+1)(x+2)}$

Ans. Let, $I = \int_1^2 \frac{dx}{(x+1)(x+2)}$

$= \int_1^2 \left[\frac{1}{x+1} - \frac{1}{x+2}\right] dx$

$= \left(\log|x+1| - \log|x+2|\right)_1^2$

$= \log\left|\frac{x+1}{x+2}\right|_1^2$

$= \log\left(\frac{3}{4}\right) - \log\left(\frac{2}{3}\right)$

$= \log\left(\frac{3}{4} \div \frac{2}{3}\right)$

$= \log\left(\frac{9}{8}\right)$

5) $\int_0^2 \frac{5x+1}{x^2+4}\,dx$

Ans. $= 5\int_0^2 \frac{x}{x^2+4}\,dx + \int_0^2 \frac{1}{x^2+4}\,dx$

$= \frac{5}{2}\int_0^2 \frac{2x}{x^2+4}\,dx + \int_0^2 \frac{1}{x^2+4}\,dx$

$= \frac{5}{2}\left[\log|x^2+4|\right]_0^2 + \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= \frac{5}{2}[\log(2^2+4) - \log 4] + \frac{1}{2}\left[\tan^{-1}\left(\frac{2}{2}\right) - 0\right]$

$= \frac{5}{2}\left[\log\left|\frac{8}{4}\right|\right] + \frac{1}{2}\left[\left|\frac{\pi}{4}\right| - 0\right]$

$= \frac{\pi}{8} + \frac{5}{2}\log 2$

6) $\int_0^1 \frac{1}{\sqrt{-(x^2-2x-3)}}\,dx$

Ans. $= \int_0^1 \frac{dx}{\sqrt{-(x^2-2x+1-4)}}$

$= \int_0^1 \frac{1}{\sqrt{2^2-(x-1)^2}}\,dx = \left[\sin^{-1}\left(\frac{x-1}{2}\right)\right]_0^1$

$= \left[0 + \frac{\pi}{6}\right] = +\frac{\pi}{6}$

7) $\int_3^5 \frac{1}{\sqrt{x+4}+\sqrt{x-2}}\,dx$

Ans. $= \int_3^5 \frac{\sqrt{x+4}-\sqrt{x-2}}{\left(\sqrt{x+4}+\sqrt{x-2}\right)\left(\sqrt{x+4}-\sqrt{x-2}\right)}\,dx$

$= \frac{1}{6}\left[\int_3^5 \sqrt{x+4} - \int_3^5 \sqrt{x-2}\,dx\right]$

$= \frac{2}{18}\left\{\left[(x+4)^{3/2}\right]_3^5 - \left[(x-2)^{3/2}\right]_3^5\right\}$

$= \frac{1}{9}\left\{\left[27 - 7\sqrt{7}\right] - \left[3\sqrt{3} - 1\right]\right\}$

$= \frac{1}{9}\left[27 - 7\sqrt{7} - 3\sqrt{3} + 1\right]$

$= \frac{1}{9}\left[28 - 7\sqrt{7} - 3\sqrt{3}\right]$

8) $\int_0^{\pi/2} \frac{dx}{1+\cos x}$

Ans. $= \frac{1}{2}\int_0^{\pi/2} \sec^2\left(\frac{x}{2}\right)$

$= \frac{1}{2}\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/2}$

$= \frac{1}{2}\times 2\left[\tan\frac{\pi}{4} - 0\right]$

$= 1$

9) $\int_{-\pi/4}^{\pi/4} \frac{dx}{1-\sin x}$

Ans. $= \int_{-\pi/4}^{\pi/4} \frac{1+\sin x}{\cos^2 x}$

$= \int_{-\pi/4}^{\pi/4} \left(\sec^2 x + \sec x \tan x\right)dx$

$= [\tan x + \sec x]_{-\pi/4}^{\pi/4}$

$= \left[\tan\frac{\pi}{4} + \sec\frac{\pi}{4}\right] - \left[-\tan\frac{\pi}{4} + \sec\frac{\pi}{4}\right]$

$= 1 + \sqrt{2} - \left(-1 + \sqrt{2}\right)$

$= 1 + \sqrt{2} + 1 - \sqrt{2}$

$= 2$

10) $\int_{\pi/3}^{\pi/2} \frac{\sin^2 x}{(1-\cos x)^2}\,dx$

Ans. $= \int_{\pi/3}^{\pi/2} \left[\frac{2\sin x/2 \, \cos x/2}{2\sin^2 x/2}\right]^2 dx$

$= \int_{\pi/3}^{\pi/2} \left[\cot^2\frac{x}{2}\right] dx$

$$= \int_{\pi/3}^{\pi/2} \left[\operatorname{cosec}^2 \frac{x}{2} - 1\right] dx$$

$$= -\left[\frac{\cot \frac{x}{2}}{\frac{1}{2}} x\right]_{\pi/3}^{\pi/2} - [x]_{\pi/3}^{\pi/2}$$

$$= -2\left[1 - \sqrt{3}\right] - \left[\frac{\pi}{2} - \frac{\pi}{3}\right]$$

$$= -2(1 - \sqrt{3}) - \frac{\pi}{6}$$

$$= -2 + 2\sqrt{3} - \frac{\pi}{6}$$

BOARD PROBLEMS

1) $\int_0^2 \left(3x^2 + 2x - 1\right) dx$

Ans. $I = \int_0^2 \left(3x^2 + 2x - 1\right) dx$

$$= 3\int_0^2 x^2\, dx + 2\int_0^2 x\, dx - \int_0^2 dx$$

$$= 3\left(x^3\right)_0^2 + \frac{2}{2}\left(x^2\right)_0^2 - (x)_0^2$$

$$= 8 + 4 - 2 = 10$$

2) $\int_0^1 \frac{x^2 + 3x + 2}{\sqrt{x}}\, dx$

Ans. $I = \int_0^1 \frac{x^2 + 3x + 2}{\sqrt{x}}\, dx$

$$= \int_0^1 \frac{x^2}{\sqrt{x}}\, dx + 3\int_0^1 \frac{x}{\sqrt{x}}\, dx + 2\int_0^1 \frac{dx}{\sqrt{x}}$$

$$= \int_0^1 x^{3/2}\, dx + 3\int_0^1 x^{1/2}\, dx + 2\int_0^1 \frac{dx}{\sqrt{x}}$$

$$= \frac{2}{5}\left(x^{5/2}\right)_0^1 + 3\times\frac{2}{3}\left(x^{3/2}\right)_0^1 + 2.2\left(\sqrt{x}\right)_0^1$$

$$= \frac{2}{5}(1) + 2(1) + 4$$

$$= \frac{2}{5} + 6$$

$$= \frac{32}{5}$$

3) $\int_0^1 \frac{x}{x+1}\, dx$

Ans. $I = \int_0^1 \frac{x}{x+1}\, dx$

$$= \int_0^1 \frac{(x+1) - 1}{x+1}\, dx$$

$$= \int_0^1 \left(1 - \frac{1}{x+1}\right) dx$$

$$= \int_0^1 dx - \int_0^1 \frac{dx}{x+1}$$

$$= (x)_0^1 - \left[\log|x+1|\right]_0^1$$

$$= 1 - \log 2$$

4) $\int_0^1 2^x\, dx$

Ans. $I = \int_0^1 2^x\, dx$

$$= \left(\frac{2^x}{\log 2}\right)_0^1$$

$$= \frac{1}{\log 2}\left(2^1 - 2^0\right)$$

$$= \frac{1}{\log 2}(2 - 1)$$

$$= \frac{1}{\log 2}$$

5) $\int_0^1 \frac{dx}{1+x^2}$

Ans. $I = \int_0^1 \frac{dx}{1+x^2}$

$= \left(\tan^{-1} x\right)_0^1$

$= \tan^{-1}(1) - 0$

$= \frac{\pi}{4}$

6) $\int_0^1 \frac{x^2}{1+x^2} dx$

Ans. $I = \int_0^1 \frac{x^2}{1+x^2} dx$

$= \int_0^1 \frac{\left(x^2+1\right)-1}{1+x^2} dx$

$= \int_0^1 dx - \int_0^1 \frac{dx}{1+x^2}$

$= (x)_0^1 - \left(\tan^{-1} x\right)_0^1$

$= 1 - \left\{\tan^{-1}(1)\right\}$

$= 1 - \frac{\pi}{4}$

7) $\int_0^1 \frac{x^2-1}{x^2+1} dx$

Ans. $I = \int_0^1 \frac{x^2-1}{x^2+1} dx$

$= \int_0^1 \frac{\left(x^2+1\right)-2}{x^2+1} dx$

$= \int_0^1 dx - 2\int_0^1 \frac{dx}{x^2+1}$

$= (x)_0^1 - 2\left(\tan^{-1} x\right)_0^1$

$= 1 - 2\left\{\tan^{-1}(1)\right\}$

$= 1 - 2.\frac{\pi}{4}$

$= 1 - \frac{\pi}{2}$

8) $\int_0^1 \frac{1-x^2}{1+x^2} dx$

Ans. $I = \int_0^1 \frac{1-x^2}{1+x^2} dx$

$= \int_0^1 \frac{2-\left(x^2+1\right)}{1+x^2} dx$

$= 2\int_0^1 \frac{dx}{1+x^2} - \int_0^1 dx$

$= 2\left(\tan^{-1} x\right)_0^1 (x)_0^1$

$= 1 - 2.\frac{\pi}{4}$

$= \frac{\pi}{2} - 1$

9) $\int_0^1 \frac{dx}{\sqrt{x+3}-\sqrt{x+1}}$

Ans. $I = \int_0^1 \frac{dx}{\sqrt{x+3}-\sqrt{x+1}}$

$= \int_0^1 \frac{\left(\sqrt{x+3}+\sqrt{x+1}\right)}{\left(\sqrt{x+3}-\sqrt{x+1}\right)\left(\sqrt{x+3}+\sqrt{x+1}\right)} dx$

$= \int_0^1 \frac{\sqrt{x+3}+\sqrt{x+1}}{(x+3)-(x+1)} dx$

$= \frac{1}{2}\left[\int_0^1 \sqrt{x+3}\, dx + \int_0^1 \sqrt{x+1}\, dx\right]$

$= \frac{1}{2} \times \frac{2}{3}\left[(x+3)^{3/2}\right]_0^1 + \frac{1}{2} \times \frac{2}{3}\left[(x+1)^{3/2}\right]_0^1$

$= \frac{1}{3}\left[4^{3/2} - 3^{3/2} + 2^{3/2} - 1^{3/2}\right]$

$= \frac{1}{3}\left[8 - 3\sqrt{3} + 2\sqrt{2} - 1\right]$

$$= \frac{1}{3}\left[7 - 3\sqrt{3} + 2\sqrt{2}\right]$$

10) $\int_0^1 \frac{2 - x^2}{1 + x^2} dx$

Ans. $I = \int_0^1 \frac{2 - x^2}{1 + x^2} dx$

$\therefore \quad I = \int_0^1 \frac{3 - \left(x^2 + 1\right)}{\left(1 + x^2\right)} dx$

$\therefore \quad I = \int_0^1 \left(\frac{3}{1 + x^2} - 1\right) dx$

$\therefore \quad I = 3\int_0^1 \frac{1}{1 + x^2} dx - \int_0^1 dx$

$\therefore \quad I = 3\left(\tan^{-1} x\right)_0^1 - (x)_0^1$

$\therefore \quad I = 3.\frac{\pi}{4} - 1$

$\therefore \quad I = \frac{3\pi}{4} - 1$

11) $\int_0^1 \frac{x + 4}{x^2 + 5} dx$

Ans. $I = \int_0^1 \frac{x + 4}{x^2 + 5} dx$

$\therefore \quad I = \int_0^1 \frac{x}{x^2 + 5} dx + 4\int_0^1 \frac{dx}{x^2 + 5}$

$\therefore \quad I = \frac{1}{2}\int_0^1 \frac{2x}{x^2 + 5} dx + 4\int_0^1 \frac{dx}{\left(x^2\right) + \left(\sqrt{5}\right)^2}$

$\therefore \quad I = \frac{1}{2}\left[\log\left|x^2 + 5\right|\right]_0^1 + 4.\frac{1}{\sqrt{5}}\left[\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_0^1$

$\therefore \quad I = \frac{1}{2}[\log 6 - \log 5] + 4.\frac{1}{\sqrt{5}}\left[\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)\right]_0^1$

$\therefore \quad I = \frac{1}{2}[\log 6 - \log 5] + \frac{4}{\sqrt{5}}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\therefore \quad I = \frac{1}{2}\log\frac{6}{5} + \frac{4}{\sqrt{5}}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

12) $\int_2^5 \frac{dx}{\sqrt{5 + 4x - x^2}}$

Ans. $I = \int_2^5 \frac{dx}{\sqrt{5 + 4x - x^2}}$

$\therefore \quad I = \int_2^5 \frac{dx}{\sqrt{9 - 4 + 4x - x^2}}$

$\therefore \quad I = \int_2^5 \frac{dx}{\sqrt{9 - \left(x^2 - 4x + 4\right)}}$

$\therefore \quad I = \left[\sin^{-1}\left(\frac{x - 2}{3}\right)\right]_2^5$

$\therefore \quad I = \sin^{-1}\left(\frac{5 - 2}{3}\right) - \sin^{-1}(0)$

$\therefore \quad I = \frac{\pi}{2}$

13) $\int_2^3 \frac{x\,dx}{(x + 3)(x + 2)}$

Ans. Let $\frac{x}{(x + 3)(x + 2)} = \frac{A}{(x + 3)} + \frac{B}{(x + 2)}$

$\therefore \quad x = A(x + 2) + B(x + 3)$

$\therefore \quad x = (A + B)x + (2A + 3B)$

Compairing co-efficients of x and constants from both sides, we have

$A + B = 1 \quad \} \times 2$

$2A + 3B = 0$

$$\begin{array}{r} 2A + 2B = 2 \\ 2A + 3B = 0 \\ - \quad - \quad\quad - \\ \hline -B = 2 \end{array}$$

$\therefore \quad B = -2$

$\therefore \quad A = 1 - B$

$\therefore \quad A = 1 - (-2)$

$\therefore \quad A = 3$

$\therefore \quad I = \int_2^3 \left(\frac{3}{x + 3} - \frac{2}{x + 2}\right) dx$

$\therefore \quad I = 3\left[\log|x+3|\right]_2^3 - 2\left[\log|x+2|\right]_2^3$

$\therefore \quad I = 3[\log 6 - \log 5] - 2[\log 5 - \log 4]$

$\therefore \quad I = 3\left[\log\frac{6}{5}\right] - 2\log\left[\frac{5}{4}\right]$

$\therefore \quad I = \log\left(\frac{6}{5}\right)^3 - \log\left(\frac{5}{4}\right)^2$

$\therefore \quad I = \log\left[\frac{6^3}{5^3} \times \frac{4^2}{5^2}\right]$

$= \log\frac{3456}{3125}$

14) $\int_1^2 \frac{dx}{x^2 - 2x + 2}$

Ans. $I = \int_1^2 \frac{dx}{x^2 - 2x + 2}$

$\therefore \quad I = \int_1^2 \frac{dx}{(x-1)^2 + 1^2}$

$\therefore \quad I = \frac{1}{1}\left[\tan^{-1}\left(\frac{x-1}{1}\right)\right]_1^2$

$\therefore \quad I = \tan^{-1} 2 - \tan^{-1} 0$

$\therefore \quad I = \tan^{-1} 2$

15) $\int_2^3 \frac{dx}{x^2 + 5x + 6}$

$\therefore \quad I = \int_2^3 \frac{dx}{x^2 + 5x + 6}$

$\therefore \quad I = \int_2^3 \frac{dx}{(x+2)(x+3)}$

$\therefore \quad I = \int_2^3 \left(\frac{1}{x+2} - \frac{1}{x+3}\right)dx$

$\therefore \quad I = \int_2^3 \frac{1}{x+2}dx - \int_2^3 \frac{dx}{x+3}$

$\therefore \quad I = \left[\log|x+2|\right]_2^3 - \left[\log|x+3|\right]_2^3$

$\therefore \quad I = [\log 5 - \log 4] - (\log 6 - \log 5)$

$\therefore \quad I = 2\log 5 - (\log 4 + \log 6)$

$\therefore \quad I = \log 5^2 - \log 24$

$\therefore \quad I = \log\frac{25}{24}$

16) $\int_0^{\pi/2} \sin^2 x \, dx$

Ans. $I = \int_0^{\pi/2} \sin^2 x \, dx$

$\therefore \quad I = \frac{1}{2}\int_0^{\pi/2} (1 - \cos 2x) \, dx$

$\therefore \quad I = \frac{1}{2}\left[\int_0^{\pi/2} dx - \int_0^{\pi/2} \cos 2x \, dx\right]$

$\therefore \quad I = \frac{1}{2}\left[(x)_0^{\pi/2} - \left(\frac{\sin 2x}{2}\right)_0^{\pi/2}\right]$

$\therefore \quad I = \frac{1}{2}\left[\frac{\pi}{2} - \frac{1}{2}(\sin\pi - \sin 0)\right]$

$\therefore \quad I = \frac{\pi}{4}$

17) $\int_{\pi/6}^{\pi/3} \sin^2 x \, dx$

Ans. $I = \int_{\pi/6}^{\pi/3} \sin^2\left(\frac{\pi}{3} + \frac{\pi}{6} - x\right) dx$ ....(i)

$I = \int_{\pi/6}^{\pi/3} \sin^2\left(\frac{\pi}{2} - x\right) dx$

$\therefore \quad I = \int_{\pi/6}^{\pi/3} \cos^2 x \, dx$ . ...(ii)

Adding (i) and (ii) we have

$\therefore \quad 2I = \int_{\pi/6}^{\pi/3} \left(\sin^2 x \cos^2 x\right) dx$

$\therefore \quad 2I = \int_{\pi/6}^{\pi/3} dx$

$\therefore \quad 2I = (x)_{\pi/6}^{\pi/3}$

$\therefore \quad 2I = \frac{\pi}{3} - \frac{\pi}{6}$

$\therefore \quad I = \frac{\pi}{12}$

18) $\int_{\pi/4}^{\pi/2} \cot^2 dx$

$\therefore \quad I = \int_{\pi/4}^{\pi/2} \left(\operatorname{cosec}^2 x - 1\right) dx$

$\therefore \quad I = \int_{\pi/4}^{\pi/2} \operatorname{cosec}^2 x\, dx - \int_{\pi/4}^{\pi/2} dx$

$\therefore \quad I = -(\cot x)_{\pi/4}^{\pi/2} - (x)_{\pi/4}^{\pi/2}$

$\therefore \quad I = -\left(\cot\frac{\pi}{2} - \cot\frac{\pi}{4}\right) - \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$

$\therefore \quad I = -(0-1) - \left(\frac{2\pi - \pi}{4}\right)$

$I = 1 - \frac{\pi}{4}$

19) $\int_{0}^{\pi/2} \frac{\sin^2 x}{(1+\cos x)^2} dx$

Ans. $I = \int_{0}^{\pi/2} \frac{\sin^2 x}{(1+\cos x)^2} dx$

$\therefore \quad I = \int_{0}^{\pi/2} \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{\left(2\cos^2\frac{x}{2}\right)^2} dx$

$\therefore \quad I = \int_{0}^{\pi/2} \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{4\cos^2\frac{x}{2}.\cos^2\frac{x}{2}} dx$

$\therefore \quad I = \int_{0}^{\pi/2} \tan^2\frac{x}{2}\, dx$

$\therefore \quad I = \int_{0}^{\pi/2} \left(\sec^2\frac{x}{2} - 1\right) dx$

$\therefore \quad I = \int_{0}^{\pi/2} \sec^2\frac{x}{2} dx - \int_{0}^{\pi/2} dx$

$\therefore \quad I = \left(\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right)_0^{\pi/2} - (x)_0^{\pi/2}$

$\therefore \quad I = \left(\tan\frac{\pi}{4}\right) - \frac{\pi}{2}$

$\therefore \quad I = 2 - \frac{\pi}{2}$

GROUP (B)-HOME WORK PROBLEMS

1) $\int_0^1 x\sqrt{1+x^2}\, dx.$

Ans. $1 + x^2 = t \qquad x = 0, t = 1$

$2x\, dx = dt \qquad x = 1, t = 2$

$\therefore \quad x\, dx = \frac{dt}{2}$

$= \frac{1}{2}\int_1^2 \sqrt{t}\, dt$

$= \frac{1}{2}\left[\frac{t^{3/2}}{3/2}\right]_1^2$

$= \frac{1}{3}\left[t^{3/2}\right]_1^2$

$= \frac{1}{3}\left[2\sqrt{2} - 1\right]$

2) $\int_0^1 x(1-x^2)^{5/2}\, dx$

Ans. $(1 - x^2) = t \qquad x = 0, \quad t = 1$

$x = \sqrt{1-t} \qquad x = 1, t = 0$

$-2x dx = dt$

$\therefore \quad I = \frac{-1}{2} \int_1^0 t^{5/2}\, dt$

$= \frac{-1}{2} \left[\frac{t^{7/2}}{7/2} \right]_1^0 = \frac{-1}{7} (0 - 1) = \frac{1}{7}$

3) $\int_0^1 \sqrt{\frac{1 - x}{1 + x}}\, dx$

Ans. $x = \cos\theta$, $\quad x = 0, \theta = \pi/2$

$dx = -\sin\theta\, d\theta \quad x = 1, \theta = 0$

$= \int_{\pi/2}^{0} \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} \times (-\sin\theta)\, d\theta$

$= \int_{\pi/2}^{0} \sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}} \times -\sin\theta\, d\theta$

$= -\int_{\pi/2}^{0} \frac{\sin\theta/2}{\cos\theta/2} \times 2\sin\theta/2 \cos\theta/2\, d\theta$

$= -2 \int_{\pi/2}^{0} \frac{1 - \cos\theta}{2} d\theta$

$= \int_{\pi/2}^{0} (\cos\theta - 1)\, d\theta$

$= [\sin\theta - \theta]_{\pi/2}^{0}$

$= -\left[1 - \frac{\pi}{2}\right] = \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$

4) $\int_0^1 \frac{1}{(1 + x^2)^2}\, dx$

Ans. $x = \tan\theta \quad \therefore\ dx = \sec^2\theta\, d\theta$

$\therefore$ When $x = 0, \theta = 0$

$x = 1, \theta = \frac{\pi}{4}$

$\therefore \quad I = \int_0^{\pi/4} \frac{1}{\sec^4\theta} \times \sec^2\theta\, d\theta$

$= \int_0^{\pi/4} \frac{1}{\sec^2\theta} d\theta = \int_0^{\pi/4} \cos^2\theta\, d\theta$

$= \frac{1}{2} \int_0^{\pi/4} (1 + \cos 2\theta)\, d\theta$

$= \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)_0^{\pi/4}$

$= \frac{1}{4}(2\theta + \sin 2\theta)_0^{\pi/4}$

$= \frac{1}{4}\left[\frac{\pi}{2} + 1 - 0\right] = \frac{\pi}{8} + \frac{1}{4}$

$= \frac{1}{4}\left(\frac{\pi}{2} + 1\right)$

5) $\int_{a/2}^{a} \frac{a^2 - x^2}{x^2}\, dx$

Ans. $x = a\sin\theta$, $\quad x = a, \theta = \pi/2$

$dx = a\cos\theta\, d\theta \quad x = \frac{a}{2}, \theta = \pi/6$

$\therefore \quad I = \int_{\pi/6}^{\pi/2} \frac{\sqrt{a^2 - a^2\sin^2\theta}}{a^2\sin^2\theta} \cdot a\cos\theta\, d\theta$

$= \int_{\pi/6}^{\pi/2} \frac{a\cos\theta}{a\sin^2\theta} \cos\theta\, d\theta$

$= \int_{\pi/6}^{\pi/2} \cot^2\theta\, d\theta$

$= \int_{\pi/6}^{\pi/2} (\operatorname{cosec}^2\theta - 1)\, d\theta$

$= [-\cot\theta]_{\pi/6}^{\pi/2} - [\theta]_{\pi/6}^{\pi/2}$

$= (0 + \sqrt{3}) - \left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= 0 + \sqrt{3} - \frac{2}{6}\pi$

$= \sqrt{3} - \frac{\pi}{3}$

6) $\int_0^3 \frac{5x}{\sqrt{x^2 + 4}}\, dx$

Ans. $x^2 + 4 = t \quad x = 0, t = 4$

$2x dx = dt \quad x = 3, t = 13$

$\therefore \quad I = \frac{5}{2} \int_4^{13} \frac{1}{\sqrt{t}}\, dt$

$= \frac{5}{2} \int_4^{13} t^{-1/2}\, dt$

$= \frac{5}{2}\left[\frac{t^{1/2}}{1/2}\right]_4^{13}$

$= 5\,[\sqrt{13} - 2]$

7) $\int_0^1 \frac{dx}{(2-x^2)\sqrt{4-x^2}}$

Ans. Let, $I = \int_0^1 \frac{dx}{(2-x^2)\sqrt{4-x^2}}$

put $x = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$,
When $x = 0 \quad \therefore \theta = 0$

when $x = 1 \quad \therefore \theta = \frac{\pi}{6}$

So,

$$I = \int_0^{\pi/6} \frac{2\cos\theta\, d\theta}{(2-4\sin^2\theta)\left(\sqrt{4-4\sin^2\theta}\right)}$$

$$= \int_0^{\pi/6} \frac{2\cos\theta\, d\theta}{2(1-2\sin^2\theta)\times 2\sqrt{1-\sin^2\theta}}$$

$$= \int_0^{\pi/6} \frac{2\cos\theta\, d\theta}{2(\cos 2\theta)\times 2\cos\theta}$$

$$= \frac{1}{2}\int_0^{\pi/6} \sec 2\theta\, d\theta$$

$$= \frac{1}{2}\left[\frac{\log|\sec 2\theta + \tan 2\theta|}{2}\right]_0^{\pi/6}$$

$$= \frac{1}{4}\log|\sec 2\theta + \tan 2\theta|\Big|_0^{\pi/6}$$

$$= \frac{1}{4}\log\left|2+\sqrt{3}\right|$$

8) $\int_0^{\pi/4} \frac{dx}{4\sin^2 x + 5\cos^2 x}$

Ans. Let $I = \int_0^{\pi/4} \frac{dx}{4\sin^2 x + 5\cos^2 x}$

Dividing Nr and Dr by $\cos^2 x$,

$\therefore \quad I = \int_0^{\pi/4} \frac{\sec^2 x\, dx}{4\tan^2 x + 5}$

put $\tan x = t \quad \therefore \sec^2 x\, dx = dt$
when $x = 0$ then $t = 0$

when $x = \frac{\pi}{4}$ then $t = 1$

So,

$$I = \int_0^1 \frac{dt}{4t^2 + 5}$$

$$= \int_0^1 \frac{dt}{(2t)^2 + (\sqrt{5})^2}$$

$$= \frac{1}{\sqrt{5}}\left[\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right)\right]_0^1 \times \frac{1}{2}$$

$$= \frac{1}{2\sqrt{5}}\left[\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

9) $\int_0^{\pi/2} \sin^2\theta \cos^2\theta \cdot \cos\theta\, d\theta$

Ans. $\int_0^{\pi/2} \sin^2\theta\,(1-\sin^2\theta)\cdot\cos\theta\, d\theta$

put $\sin\theta = t$, $\theta = 0$, $t = 0$

$\cos\theta\, d\theta = dt$, $\theta = \frac{\pi}{2}$, $t = 1$

$\therefore \quad I = \int_0^1 t^2(1-t^2)\, dt$

$$= \left[\frac{t^3}{3} - \frac{t^5}{5}\right]_0^1$$

$$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

BOARD PROBLEMS

1) $\int_0^{\pi/2} \sin^2 x \cos x\, dx$

Ans. Let $\sin x = z$
$\cos x\, dx = dz$

When $x = \frac{\pi}{2}$, $z = 1$
$x = 0$, $z = 0$

$\therefore \quad I = \int_0^1 z^2\,dz$

$\therefore \quad I = \left(\frac{z^3}{3}\right)_0^1$

$= \frac{1}{3}$

2) $\int_0^{\pi/2} \sin^4 x\ \cos^3 x\,dx$

Ans. $I = \int_0^{\pi/2} \sin^4 x\ \cos^3 x\,dx$

$= \int_0^{\pi/2} \sin^4 x\ \left(1 - \sin^2 x\right).\cos x\,dx$

Let sinx = z

cos x dx = dz

When $x = \frac{\pi}{2}$, z = 1

x = 0 , z = 0

$\therefore \quad I = \int_0^1 z^4\left(1 - z^2\right)dz$

$\therefore \quad I = \int_0^1 \left(z^4 - z^6\right)dz$

$\therefore \quad I = \int_0^1 z^4 dz - \int_0^1 z^6 dz$

$\therefore \quad I = \left(\frac{z^5}{5}\right)_0^1 - \left(\frac{z^7}{7}\right)_0^1$

$\therefore \quad I = \frac{1}{5} - \frac{1}{7}$

$\therefore \quad I = \frac{2}{35}$

3) $\int_0^{\pi/2} \cos^3 x\,dx$

Ans. $I = \int_0^{\pi/2} \cos^3 x\,dx$

$\therefore \quad I = \frac{1}{4}\int_0^{\pi/2} (3\cos x + \cos 3x)\,dx$

$\therefore \quad I = \frac{1}{4}\left[3\int_0^{\pi/2} \cos x\,dx + \int_0^{\pi/2} \cos 3x\,dx\right]$

$\therefore \quad I = \frac{1}{4}\left[3(\sin x)_0^{\pi/2} + \left(\frac{\sin 3x}{3}\right)_0^{\pi/2}\right]$

$\therefore \quad I = \frac{1}{4}\left[3(1-0) + \frac{1}{3}(-1)\right]$

$\therefore \quad I = \frac{1}{4}\left[3 - \frac{1}{3}\right]$

$\therefore \quad I = \frac{1}{4}\left[\frac{8}{3}\right]$

$= \frac{2}{3}$

4) $\int_0^{\pi/2} \frac{\cos x}{1 + \sin x}\,dx$

Ans. $I = \int_0^{\pi/2} \frac{\cos x}{1 + \sin x}\,dx$

let 1 + sinx = z

cos x dx = dz

When $x = \frac{\pi}{2}$, z = 2

x = 0 , z = 1

$\therefore \quad I = \int_1^2 \frac{dz}{z}$

$\therefore \quad I = \left[\log|z|\right]_1^2$

$\therefore \quad I = \log 2$

5) $\int_0^{\pi/2} \frac{\sin x}{1+\cos x} dx$

Ans. Let 1 + cos x = z

– sin x dx = dz

∴ sin x dx = – dz

When $x = \frac{\pi}{2}$, z = 1

x = 0 , z = 2

∴ $I = -\int_2^1 \frac{dz}{z}$

∴ $I = -[\log|z|]_2^1$

∴ $I = -[\log 1 - \log 2]$

∴ I = log 2

6) $\int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)^3}$

Ans. Let 1 + sin x = z

cos x dx = dz

When $x = \frac{\pi}{2}$, z = 2

x = 0 , z = 1

∴ $I = \int_1^2 \frac{dz}{z^3}$

∴ $I = \left(\frac{z^{-2}}{-2}\right)_1^2$

∴ $I = -\frac{1}{2}\left(\frac{1}{z^2}\right)_1^2$

∴ $I = -\frac{1}{2}\left(\frac{1}{4}-1\right)$

∴ $I = -\frac{1}{2}\left(-\frac{3}{4}\right)$

∴ $I = \frac{3}{8}$

7) $\int_0^{\pi/2} \frac{\sin x}{(1+\cos x)^3} dx$

Ans.

Let 1 + cos x = z

sin x dx = dz

When $x = \frac{\pi}{2}$, z = 1

x = 0 , z = 2

∴ $I = \int_2^1 \frac{-dz}{z^3}$

∴ $I = -\int_2^1 z^{-3}\, dz$

∴ $I = -\left(\frac{1}{-2z^2}\right)_2^1$

∴ $I = \frac{1}{2}\left(1-\frac{1}{4}\right)$

∴ $I = \frac{1}{2}\left(\frac{3}{4}\right)$

$= \frac{3}{8}$

8) $\int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$

Ans. $I = \int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$

∴ $I = \int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(1++\sin x+1)}$

Let 1 + sin x = z

cos x dx = dz

When $x = \frac{\pi}{2}$, z = 2

x = 0 , z = 1

∴ $I = \int_1^2 \frac{dz}{z(z+1)}$

$= \int_1^2 \left(\frac{1}{z} - \frac{1}{z+1}\right) dz$

$= \int_1^2 \frac{1}{z} dz - \int_1^2 \frac{dz}{z+1}$

$= [\log|z|]_1^2 - [\log|z+1|]_1^2$ s

= log 2 – (log 3 – log 2)

= 2 log 2 – log 3

$= \log \frac{4}{3}$

9) $\int_0^{\pi} \frac{\sin x\, dx}{1+\cos^2 x}$

Ans. Let cos x = z

sin x dx = dz

When $x = \frac{\pi}{2}$, z = 0

x = 0 , z = 1

$\therefore \quad I = \int_1^0 \frac{-dz}{1+z^2}$

$\therefore \quad I = -\left[\tan^{-1}(z)\right]_1^0$

$\therefore \quad I = -\left[0 - \frac{\pi}{4}\right]$

$\therefore \quad I = \frac{\pi}{4}$

10) $\int_0^{\pi/2} \frac{dx}{2\cos^2 \frac{x}{2}}$

Ans. $I = \int_0^{\pi/2} \frac{dx}{2\cos^2 \frac{x}{2}}$

$\therefore \quad I = \frac{1}{2} \int_0^{\pi/2} \sec^2 \frac{x}{2} dx$

$\therefore \quad I = \frac{1}{2} \cdot \left(\frac{\tan \frac{x}{2}}{\frac{1}{2}} \right)_0^{\pi/2}$

$\therefore \quad I = 2 \cdot \frac{1}{2} \cdot \left(\tan \frac{x}{2} \right)_0^{\pi/2}$

$\therefore \quad I = \tan \frac{\pi}{4} - \tan 0$

$\therefore \quad I = 1$

11) $\int_0^{\pi/2} \frac{dx}{5+4\cos x}$

Ans. $I = \int_0^{\pi/2} \frac{dx}{5+4\cos x}$

$\therefore \quad I = \int_0^{\pi/2} \frac{dx}{5+4\left(\frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}} \right)}$

$\therefore \quad I = \int_0^{\pi/2} \frac{\left(1+\tan^2 \frac{x}{2}\right) dx}{5\left(1+\tan^2 \frac{x}{2}\right) + 4\left(1-\tan^2 \frac{x}{2}\right)}$

$\therefore \quad I = \int_0^{\pi/2} \frac{\sec^2 \frac{x}{2} dx}{\tan^2 \frac{x}{2} + 9}$

Let $\tan \frac{x}{2} = z$

$\sec^2 \frac{x}{2}\, dx = 2\, dz$

When $x = \frac{\pi}{2}$, z = 1

x = 0 , z = 0

$\therefore \quad I = \int_0^1 \frac{2dz}{z^2 + (3)^2}$

$\therefore \quad I = 2\int_0^1 \frac{dz}{z^2 + 3^2}$

$\therefore \quad I = 2 \cdot \frac{1}{3} \left(\tan^{-1} \frac{z}{3} \right)_0^1$

$\therefore \quad I = \frac{2}{3} \left(\tan^{-1} \frac{1}{3} \right)$

12) $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

Ans. Let $\tan^{-1} x = z$

$\frac{1}{1+x^2} dx = dz$

When x = 1 , $z = \frac{\pi}{2}$

x = 0 , z = 0

$\therefore \quad I = \int_0^{\pi/2} z\, dz$

$\therefore \quad I = \frac{1}{2}\left(z^2\right)_0^{\pi/4}$

$I = \frac{\pi^2}{32}$

13) $\int_0^2 \frac{e^{1/x}}{x^2}\,dx$

Ans. $I = \int_0^2 \frac{e^{1/x}}{x^2}\,dx$

Let $\frac{1}{x} = z$

$\therefore \quad \frac{1}{x^2}dx = -dz$

When $x = 2$, $z = \frac{1}{2}$

$x = 0$, $z = \infty$

$\therefore \quad I = \int_{\infty}^{1/2} e^z\,dz$

$\therefore \quad I = -\left(e^z\right)_{\infty}^{1/2}$

$\therefore \quad I = \left(\sqrt{e} - 0\right)$

$\therefore \quad I = -\sqrt{e}$

14) $\int_0^1 \sqrt{1-x^2}\,dx$

Ans. Let $x = \sin\theta$

$\therefore \quad dx = \cos\theta\,d\theta$

$x = 1$, $\theta = \frac{\pi}{2}$

$x = 0$, $\theta = 0$

$\therefore \quad I = \int_0^{\pi/2} \cos\theta.\cos\theta\,d\theta$

$I = \int_0^{\pi/2} \cos\theta.\cos\theta\,d\theta$

$\therefore \quad I = \int_0^{\pi/2} \cos^2\theta\,d\theta$

$\therefore \quad I = \int_0^{\pi/2} (1+\cos 2\theta)\,d\theta$

$\therefore \quad I = \frac{1}{2}\left[\int_0^{\pi/2} d\theta + \int_0^{\pi/2} \cos 2\theta\,d\theta\right]$

$\therefore \quad I = \frac{1}{2}\left[(\theta)_0^{\pi/2} + \left(\frac{\sin 2\theta}{2}\right)_0^{\pi/2}\right]$

$\therefore \quad I = \frac{1}{2}\left[\frac{\pi}{2} + \frac{1}{2}\left(\frac{\sin 2\theta}{2}\right)_0^{\pi/2}\right]$

$\therefore \quad I = \frac{1}{2}\left[\frac{\pi}{2} + \frac{1}{2}(\sin\pi - 0)\right]$

$\therefore \quad I = \frac{1}{2}\left(\frac{\pi}{2}\right)$

$\therefore \quad I = \frac{\pi^2}{4}$

15) $\int_2^3 \frac{dx}{x\left(x^3-1\right)}$

Ans. $I = \int_2^3 \frac{dx}{x\left(x^3-1\right)}$

$\therefore \quad I = \int_2^3 \frac{x^3 - \left(x^3-1\right)}{x\left(x^3-1\right)}\,dx$

$\therefore \quad I = \int_2^3 \frac{x^2}{x^3-1}\,dx - \int_2^3 \frac{1}{x}\,dx$

$I = I_1 - I_2$

$I_1 = \int_2^3 \frac{x^2}{x^3-1}\,dx$

Let $x^3 - 1 = z$,

$3x^2\,dx = dz$

$x^2\,dx = \frac{dz}{3}$,

where $x = 3$, $z = 26$

$x = 2$ $z = 7$

$\therefore \quad I_1 = \frac{1}{3}\int_7^{26} \frac{dz}{z}$

$\therefore \quad I_1 = \frac{1}{3}\left[\log|z|\right]_7^{26}$

$\therefore \quad I1 = \frac{1}{3}[\log 26 - \log 7]$

$= \frac{1}{3}\log\left(\frac{26}{7}\right)$

$\therefore \quad I_2 = \int_2^3 \frac{1}{x}dx$

$\therefore \quad I_2 = \left[\log|x|\right]_2^3$

$\therefore \quad I_2 = \log 3 - \log 2$

$\therefore \quad I_2 = \log\left[\frac{3}{2}\right]$

$\therefore \quad I_2 = I_1 - I_2$

$I = \frac{1}{3}\log\frac{26}{7} - \log\frac{3}{2}$

$I = \frac{1}{3}\log\frac{26}{7} - \frac{1}{3}.3\log\frac{3}{2}$

$I = \frac{1}{3}\left[\log\frac{26}{7} - \log\left(\frac{3}{2}\right)^3\right]$

$I = \frac{1}{3}\left[\log\frac{26}{7} - \log\frac{27}{8}\right]$

$I = \frac{1}{3}\log\left(\frac{26}{7}\times\frac{8}{27}\right)$

$I = \frac{1}{3}\log\left(\frac{208}{189}\right)$

GROUP (C)-HOME WORK PROBLEMS

1) $\int_0^\pi \sin 2x\, e^x\, dx$

Ans. Let $I = \int_0^\pi \sin 2x\, e^x\, dx$

$I = \left[\sin 2x \int e^x\, dx\right]_0^{\pi/2}$

$-\int_0^\pi \left[\int e^x\, dx \times \frac{d}{dx}(\sin 2x)\right]dx$

$I = \left[\sin 2x . e^x\right]_0^\pi - \int_0^\pi e^x \cos 2x \times 2\, dx$

$I = \left[\sin 2\pi . e^\pi - \sin 0 . e^0\right] - 2\int_0^\pi \cos 2x .\, e^x\, dx$

$I = 0 - 2\int_0^\pi \cos 2x . e^x\, dx$

$I = -2\left[\left(\cos 2x . e^x\right)^\pi - \int_0^\pi -2\sin 2x . e^x dx\right]$

$I = -2\left[\cos 2\pi . e^\pi - \cos 0 . e^0\right] - 4\int_0^\pi \sin 2x\; e^x\, dx$

$I + 4I = -2\,[e^\pi - 1]$

$5I \quad = 2(1 - e^\pi)$

$\therefore \quad I = \frac{2}{5}\,(1 - e^\pi)$

2) $\int_0^{\pi/2} e^{2x}\cos x\, dx$

Ans. Let, $I = \int_0^{\pi/2} e^{2x}\cos x\, dx$

$= \int_0^{\pi/2} \cos x . e^{2x}\, dx$

$= \left[\cos x \int e^{2x}\, dx\right]_0^{\pi/2}$

$-\int_0^{\pi/2}\left[\int e^{2x}\, dx \frac{d}{dx}(\cos x)\right]dx$

$= \frac{1}{2}\left[\cos x . e^{2x}\right]_0^{\pi/2} - \int_0^{\pi/2}\left[\int \frac{e^{2x}}{2} \times -\sin x\right]dx$

$= \frac{1}{2}\left[\cos x . e^{2x}\right]_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2} \sin x . e^{2x}\, dx$

$= \frac{1}{2}\left[\cos\frac{\pi}{2}e^\pi - \cos 0 . e^0\right] +$

$\frac{1}{2}\left[\sin x \int_0^{\pi/2} e^{2x}\, dx - \int_0^{\pi/2}\left[\frac{e^{2x}}{2}\cos x\right]dx\right]$

$\therefore \quad I = \frac{1}{2}[0 - 1] +$

$\frac{1}{2}\frac{\left[\sin x . e^{2x}\right]_0^{\pi/2}}{2} - \frac{1}{4}\int_0^{\pi/2} e^{2x}\cos x\, dx$

$\therefore \quad I + \frac{1}{4} I = \frac{-1}{2} + \frac{1}{4}\left[\sin\frac{\pi}{2} e^{\pi} - \sin 0 . e^{0}\right]$

$\therefore \quad \frac{5I}{4} = \frac{-1}{2} + \frac{1}{4}\left[e^{\pi}\right]$

$\therefore \quad \frac{5I}{4} = \frac{-2 + e^{\pi}}{4}$

$\therefore \quad I = \frac{1}{5}\left(e^{\pi} - 2\right)$

3) $I = \log x \int_1^4 x^2 \, dx - \int_1^4 \frac{x^2}{x} \, dx$

Ans. $I = \int_1^4 x^2 \log x \, dx$

$= \left[\log x \int x^2 \, dx\right]_1^4 - \int_1^4 \left(\frac{1}{x} . \int x^2 \, dx\right) dx$

$= \left[\log x . \frac{x^3}{3}\right]_1^4 - \left[\int \frac{1}{x} . \frac{x^3}{3} dx\right]_1^4$

$= \left(\frac{64}{3}\log 4 - \frac{1}{3}.0\right) - \frac{1}{3}\int_1^4 x^2 \, dx$

$= \frac{64}{3}\log 4 - \frac{1}{3}\left(\frac{x^3}{3}\right)_1^4$

$= \frac{64}{3}\log 4 - \frac{1}{9}(64 - 1)$

$= \frac{64}{3}\log 4 - 7$

4) $\int_0^{\pi/4} \sec^3 x \, dx$

Ans. Let, $I = \int_0^{\pi/4} \sec^3 x \, dx$

$= \int_0^{\pi/4} \sec x \sec^2 x \, dx$

$= \int_0^{\pi/4} \sqrt{1 + \tan^2 x} \sec^2 x \, dx$

put $\tan x = t$, $\therefore$ $\sec^2 x \, dx = dt$

when $x = 0$ then $t = 0$

when $x = \frac{\pi}{4}$ then $t = 1$

So,

$I = \int_0^1 \sqrt{1 + t^2} \, dt$

$= \left[\frac{t}{2} \times \sqrt{1 + t^2} + \frac{1}{2}\log\left|t + \sqrt{1 + t^2}\right|\right]_0^1$

$= \left[\frac{1}{2}\sqrt{2} + \frac{1}{2}\log\left|1 + \sqrt{2}\right|\right] - [0]$

$= \frac{1}{\sqrt{2}} + \frac{1}{2}\log\left(1 + \sqrt{2}\right)$

5) $\int_1^e (\log x)^2 \, 1 dx$

Ans. $= (\log x)^2 \int_1^e 1 dx - \int_1^e 2 \log x \frac{1}{x} . x \, dx$

$= \left[x(\log x)^2\right]_1^e - 2\left[\log x . x - \int \frac{1}{x} . x \, dx\right]_1^e$

$= \left[x (\log x)^2\right]_1^e - 2\left[(x \log x)_1^e - [x]_1^e\right]$

$= e - 2e + 2e - 2$

$= e - 2$

6) $\int_0^1 x^3 \tan^{-1} x \, dx$

Ans. $I = \tan^{-1} x \int_0^1 x^3 \, dx -$

$\int_0^1 \left\{\frac{d}{dx}(\tan^{-1} x) \int x^3 \, dx\right\} dx$

$= \frac{[x^4 \tan^{-1} x]_0^1}{4} - \frac{1}{4}\int_0^1 \frac{x^4}{1 + x^2}$

$= \frac{1}{4} \times \frac{\pi}{4} - \frac{1}{4}\int_0^1 \frac{x^4 - 1 + 1}{1 + x^2} dx$

$\therefore \quad I = \frac{\pi}{16} - \frac{1}{4}\int_0^1 \left\{x^2 - 1 + \frac{1}{x^2 + 1}\right\} dx$

$= \frac{\pi}{16} - \frac{1}{4}\left\{\frac{\left[x^3\right]_0^1}{3} - [x]_0^1 + \left[\tan^{-1} x\right]_0^1\right\}$

$= \frac{\pi}{16} - \frac{1}{4}\left\{\frac{1}{3} - 1 + \frac{\pi}{4}\right\}$

$= \frac{\pi}{16} - \frac{1}{12} + \frac{1}{4} - \frac{\pi}{16}$

$= \frac{1}{6}$

7) $\int_0^1 x \frac{\tan^{-1} x}{(1+x^2)\sqrt{1+x^2}} dx$

Ans. put $\tan^{-1}x = t$

$\frac{1}{1+x^2} dx = dt$

$\therefore$ $x = \tan t$

when $x = 0, t = 0$

$x = 1, t = \frac{\pi}{4}$

$= \int_0^{\pi/4} \frac{t \tan t}{\sqrt{1+\tan^2 t}} dt$

$= \int_0^{\pi/4} \frac{\frac{t \sin t}{\cos t}}{\frac{1}{\cos t}} dt$

$= \int_0^{\pi/4} t \sin t \, dt$

$= \left[t\int \sin t\right]_0^{\pi/4} - \int_0^{\pi/4} \left[\int \sin t \, dt\right] dt$

$= -[t \cos t]_0^{\pi/4} s + \int_0^{\pi/4} \cos t \, dt$

$= -[t \cos t]_0^{\pi/4} + [\sin t]_0^{\pi/4}$

$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{4}\right)$

8) $\int_0^{1/\sqrt{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

Ans. Let, $\sin^{-1}x = \pi$, when $x = \frac{1}{\sqrt{2}}$, $\pi = \frac{\pi}{4}$

$\therefore$ $\sin \pi = x$ $\quad x = 0, \pi = 0$

$\therefore$ $\frac{1}{\sqrt{1-x^2}} dx = d\pi$

$\therefore$ $I = \int_0^{\pi/4} \pi \sin \pi . d\pi$

$= \left[\pi \int \sin \pi . d\pi\right]_0^{\pi/4} - \left[\int \left(1 . \int \sin \pi d\pi\right) d\pi\right]_0^{\pi/4}$

$= -(\pi \cos \pi)_0^{\pi/4} - \left[\int -\cos\pi \, d\pi\right]_0^{\pi/4}$

$= -\left(\frac{\pi}{4} . \frac{1}{\sqrt{2}}\right) + (\sin \pi)_0^{\pi/4}$

$= -\frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\left(1 - \frac{\pi}{4}\right)$

9) $\int_0^{1/\sqrt{2}} \frac{x^2 \sin^{-1} x}{\sqrt{1-x^2}} dx$

Ans. Let $t = \sin^{-1}x$, $x = \sin t$

$dt = \frac{1}{\sqrt{1-x^2}} dx$

when $x = 0, t = 0$

$x = \frac{1}{\sqrt{2}}, \frac{\pi}{4} = t$

So,

$I = \int_0^{\pi/4} \sin^2 t \cdot t \cdot dt$

$I = \int_0^{\pi/4} \left(t \sin^2 t\right) dt$

$= \int_0^{\pi/4} \frac{t(1 - \cos 2t)}{2} dt$

$= \frac{1}{2}\left[\int_0^{\pi/4} t - \int_0^{\pi/4} t \cos 2t \, dt\right]$

$= \frac{1}{2}\left[\left(\frac{t^2}{2}\right)_0^{\pi/4} - \left\{t\left(\frac{\sin 2t}{2}\right) - \int\left(1 . \frac{\sin 2t}{2}\right) dt\right\}_0^{\pi/4}\right]$

$= \frac{1}{2}\left[\frac{\pi^2}{32} - \frac{1}{2}\left\{t \sin 2t + \frac{\cos 2t}{2}\right\}_0^{\pi/4}\right]$

$= \frac{1}{4}\left[\frac{\pi^2}{16} - \left(t \sin 2t + \frac{\cos 2t}{2}\right)_0^{\pi/4}\right]$

$$=\frac{1}{4}\left[\frac{\pi^2}{16}-\left(\frac{\pi}{4}.1+\frac{1}{2}.0-0-\frac{1}{2}\right)\right]$$

$$=\frac{1}{4}\left[\frac{\pi^2}{16}-\frac{\pi}{4}+\frac{1}{2}\right]$$

$$=\frac{\pi^2}{64}-\frac{\pi}{16}+\frac{1}{8}$$

10) $\int_0^1 \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)dx$

Ans. Let, $I = \int_0^1 \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)dx$

put $x = \sin\theta$

$\therefore$ $dx = \cos\theta\, d\theta$

when x = 0 then $\theta = 0$

when x = 1 then $\theta = \pi/2$

So,

$$I = \int_0^{\pi/2} \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} \tan^{-1}(\tan\theta)\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} \theta\cos\theta\, d\theta$$

$$=\left[\theta\int\cos\theta\, d\theta\right]_0^{\pi/2}-\left[\int\left(1.\int\cos\theta\, d\theta\right)d\theta\right]_0^{\pi/2}$$

$$=(\theta.\sin\theta)_0^{\pi/2}-\left[\int\sin\theta\, d\theta\right]_0^{\pi/2}$$

$$=\left(\frac{\pi}{2}.1-0\right)-(-\cos\theta)_0^{\pi/2}$$

$$=\frac{\pi}{2}+(\cos\theta)_0^{\pi/2}$$

$$=\frac{\pi}{2}+(0-1)$$

$$=\frac{\pi}{2}-1$$

BOARD PROBLEMS

1) $\int_0^1 x e^x dx$

Ans. $I = \int_0^1 x e^x dx$

$$=\left[x\int e^x dx\right]_0^1 \left[\left\{1.\int e^x dx\right\}dx\right]_0^1$$

$$=\left(xe^x\right)_0^1-\left[\int e^x dx\right]_0^1$$

$$=\left(xe^x\right)_0^1-\left(e^x\right)_0^1$$

$= e - (e - e^0)$

$= e - e + 1$

$= 1$

2) $\int_0^2 x\, e^x\, dx$

Ans. $I = \int_0^2 x\, e^x\, dx$

$$=\left[x\int e^x dx\right]_0^2-\left[\left\{1.\int e^x dx\right\}dx\right]_0^2$$

$$=\left[x\, e^x\right]_0^2-\left[\int e^x dx\right]_0^2$$

$$=\left(x\, e^x\right)_0^2-\left(e^x\right)_0^2$$

$= 2e^2 - (e^2 - e^0)$

$= 2e^2 - e^2 + 1$

$= e^2 + 1$

3) $\int_0^{\pi/2} x\sin x\, dx$

Ans. $I = \int_0^{\pi/2} x\sin x\, dx$

$\therefore$ $I = \left[x\int \sin x\, dx\right]_0^{\pi/2} - \left[\int 1\left(\int \sin x\, dx\right)dx\right]_0^{\pi/2}$

$\therefore$ $I = \left[x(-\cos x)\right]_0^{\pi/2} - \int_0^{\pi/2} -\cos x\, dx$

$\therefore$ $I = -(x\cos x)_0^{\pi/2} - \int_0^{\pi/2} -\cos x\, dx$

$\therefore$ $I = -(x\cos x)_0^{\pi/2} + \int_0^{\pi/2} \cos x\, dx$

$\therefore \quad I = -\left(\frac{\pi}{2}.0 - 0.1\right) + (\sin x)_0^{\pi/2}$

$\therefore \quad I = 0 + (1 - 0)$

$\therefore \quad I = 1$

4) $\int_0^{\pi/2} \frac{x\,dx}{1+\cos x}$

Ans. $I = \int_0^{\pi/2} \frac{x\,dx}{1+\cos x} = \int_0^{\pi/2} \frac{x}{2\cos^2\frac{x}{2}}dx$

$= \frac{1}{2}\int_0^{\pi/2} x\sec^2\frac{x}{2}dx$

$= \frac{1}{2}\left[\left\{x\int \sec^2\frac{x}{2}dx\right\}_0^{\pi/2} - \left\{\int 1\left(\sec^2\frac{x}{2}dx\right)dx\right\}_0^{\pi/2}\right]$

$= \frac{1}{2}\left[\left(x\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right)_0^{\pi/2} - \left(\int \frac{\tan\frac{x}{2}}{\frac{1}{2}}dx\right)_0^{\pi/2}\right]$

$= \frac{1}{2}\left[2\left(x\tan\frac{x}{2}\right)_0^{\pi/2} - 2\left(\int \tan\frac{x}{2}dx\right)_0^{\pi/2}\right]$

$\therefore \quad I = \frac{1}{2}\left[2\left(x\tan\frac{x}{2}\right)_0^{\pi/2} - 2\left(\frac{\log\left|\sec\frac{x}{2}\right|}{\frac{1}{2}}\right)_0^{\pi/2}\right]$

$= \left[\left(\frac{\pi}{2} - 1 - 0\right) - 2\left(\log\sec\frac{\pi}{4} - \log\sec 0\right)\right]$

$= \frac{\pi}{2} - 2.\log\sqrt{2}$

$= \frac{\pi}{2} - 2.\frac{1}{2}\log 2$

$= \frac{\pi}{2} - \log 2$.

5) $\int_0^2 x\log x\,dx$

Ans. $I = \int_0^2 x\log x\,dx$

$\therefore \quad I = \left[\log x.\int x\,dx\right]_0^1 - \left[\int\left(\frac{1}{x}.\int x\,dx\right)dx\right]_0^2$

$\therefore \quad I = \left[\log x.\frac{x^2}{2}\right]_0^2 - \left[\int\left(\frac{1}{x}.\frac{x^2}{2}\right)dx\right]_0^2$

$\therefore \quad I = \left[\frac{x^2}{2}\log x\right]_0^2 - \frac{1}{2}\int_0^2 x\,dx$

$\therefore \quad I = 2\log 2 - \frac{1}{2}.\left(\frac{x^2}{2}\right)_0^2$

$\therefore \quad I = 2\log 2 - \frac{1}{4}(4)$

$\therefore \quad I = 2\log 2 - 1$

6) $\int_1^2 \log x\,dx$

Ans. $I = \int_1^2 \log x\,dx$

$= \left[\log x\int dx\right]_1^2 - \left[\int\left(\frac{1}{x}.\int dx\right)dx\right]_1^2$

$= (x\log x)_1^2 - \left[\int \frac{1}{x}.x\,dx\right]_1^2$

$= (2\log 2 - 1\log 1) - \left(\int dx\right)_1^2$

$= 2\log 2 - (x)^2$

$= 2\log 2 - (2-1)$

$= 2\log 2 - 1$

7) $\int_1^e \log x\,dx$

Ans. $= \int_1^e \log x.1\,dx$

$= \left[\log x\int dx\right]_1^e - \left[\int\left(\frac{1}{x}.\int dx\right)dx\right]_1^e$

$$= (x \log x)_1^e - \left[\int \left(\frac{1}{x}.x\right) dx\right]_1^e$$

$$= e \log e - \left(\int dx\right)_1^e$$

$$= e - (x)_1^e$$

$$= e - (e - 1)$$

$$= e - e + 1 = 1$$

8) $\int_0^1 x \tan^{-1} x \, dx$

Ans. $= \left[\tan^{-1} x \int x\, dx\right]_0^1 - \left[\int \left(\frac{1}{1+x^2}.\int x\, dx\right) dx\right]_0^1$

$$= \left(\tan^{-1} x . \frac{x^2}{2}\right)_0^1 - \left[\int \frac{x^2}{2\left(1+x^2\right)}\right]_0^1$$

$$= \frac{1}{2}\left(x^2 \tan^{-1} x\right)_0^1 - \frac{1}{2}\left[\int \frac{x^2 + 1 - 1}{1+x^2} dx\right]_0^1$$

$$= \frac{1}{2}\left(1 . \frac{\pi}{4}\right) - \frac{1}{2}\left[\int dx - \int \frac{dx}{1+x^2}\right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[x - \tan^{-1} x\right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[1 - \frac{\pi}{4}\right]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

GROUP (D)-HOME WORK PROBLEMS

1) $\int_0^2 x^2 \sqrt{2-x}\; dx$

Ans. Replace x by $2 - x$

$\therefore$ $I = \int_0^2 (2-x)^2 \sqrt{2-(2-x)}\; dx$

$$= \int_0^2 \left(4 - 4x + x^2\right) x^{1/2}\; dx$$

$$= \int_0^2 \left(4x^{1/2} - 4^{3/2} + x^{5/2}\right) dx$$

$$= \left[\frac{4x^{3/2}}{\frac{3}{2}} - \frac{4x^{5/2}}{\frac{5}{2}} + \frac{x^{7/2}}{\frac{7}{2}}\right]_0^2$$

$$\frac{8}{2}\left[2^{3/2}\right] - \frac{8}{5} 2^{5/2} + \frac{2}{7}\left(2^{7/2}\right)$$

$$= \frac{8}{2}\left[2\sqrt{2}\right] - \frac{8}{5}\left[4\sqrt{2}\right] + \frac{2}{7}\left[8\sqrt{2}\right]$$

$$= \frac{16\sqrt{2}}{3} - \frac{32\sqrt{2}}{5} + \frac{16\sqrt{2}}{7}$$

$$= \frac{560\sqrt{2} - 672\sqrt{2} + 240\sqrt{2}}{105}$$

$$= \frac{128\sqrt{2}}{105}$$

2) $\int_0^a x^2 (a-x)^{3/2}\, dx$

Ans. Replace x by $a - x$

$$I = \int_0^a (a-x)^2 \; \left[a - (a-x)\right]^{3/2} dx$$

$$= \int_0^a (a-x)^2 \; x^{3/2}\, dx$$

$$= \int_0^a \left(a^2 - 2ax + x^2\right) x^{3/2}\, dx$$

$$= \int_0^a \left(a^2 x^{3/2} - 2ax^{5/2} + x^{7/2}\right) dx$$

$$= \left[a^2 \frac{x^{5/2}}{\frac{5}{2}} - \frac{2ax^{7/2}}{\frac{7}{2}} + \frac{x^{9/2}}{\frac{9}{2}}\right]_0^a$$

$$= \frac{2}{5} a^2 a^{5/2} - \frac{4}{7} a\, a^{7/2} + \frac{2}{9} a^{9/2}$$

$$= a^{9/2}\left[\frac{2}{5} - \frac{4}{7} + \frac{2}{9}\right]$$

$$= a^{9/2}\left[\frac{14-20}{35} + \frac{2}{9}\right]$$

$$= a^{9/2}\left[\frac{2}{9} - \frac{6}{35}\right]$$

$$= a^{9/2}\left[\frac{70-54}{315}\right]$$

$$= \frac{16a^{9/2}}{315}$$

3) $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$

Ans. $I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\cot x}}$

$$I = \int_0^{\pi/2} \frac{dx}{1+\frac{\sqrt{\cot x}}{\sqrt{\sin x}}}$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx \quad \text{.... (i)}$$

Replace x by $\frac{\pi}{2} - x$

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\, dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\, dx \quad \text{....(ii)}$$

Adding (i) and (ii)

$$\therefore \quad 2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right) dx$$

$$\therefore \quad 2I = \int_0^{\pi/2} dx$$

$$\therefore \quad 2I = [x]_0^{\pi/2}$$

$$\therefore \quad 2I = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

4) $\int_0^{\pi/2} \frac{\sin\theta\, d\theta}{(\sin\theta+\cos\theta)^2}$

Ans. $I = \int_0^{\pi/2} \frac{\sin\theta\, d\theta}{(\sin\theta+\cos\theta)^2}$ ....(i)

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-\theta\right) d\theta}{\left[\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)\right]^2}$$

$$I = \int_0^{\pi/2} \frac{\cos\theta\, d\theta}{(\cos\theta+\sin\theta)^2} \quad \text{...(ii)}$$

Adding (i) and (ii)

$$\therefore \quad 2I = \int_0^{\pi/2} \frac{(\sin\theta+\cos\theta)\, d\theta}{(\sin\theta+\cos\theta)^2}$$

$$\therefore \quad 2I = \int_0^{\pi/2} \frac{1\, d\theta}{\sin\theta+\cos\theta}$$

$$\therefore \quad 2I = \int_0^{\pi/2} \frac{1\, d\theta}{1\sin\theta+1\cos\theta}$$

$1 = r\cos\alpha$; $1 = r\sin\alpha$

$r = \sqrt{a^2+b^2}$

$r = \sqrt{1^2+1^2}$

$r = \sqrt{2}$

$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

$$2I = \int_0^{\pi/2} \frac{d\theta}{r\cos\alpha\sin\theta + r\sin\alpha\cos\theta}$$

$$2I = \int_0^{\pi/2} \frac{d\theta}{r[\sin\alpha\cos\theta+\cos\alpha\sin\theta]}$$

$$2I = \int_0^{\pi/2} \frac{1\, d\theta}{r\sin[\alpha+\theta]} = \int_0^{\pi/2} \frac{1}{r}\operatorname{cosec}(\alpha+\theta)\, d\theta$$

$$2I = \frac{1}{r}\log\left|\operatorname{cosec}(\alpha+\theta) - \cot(\alpha+\theta)\right|_0^{\pi/2}$$

$$2I = \frac{1}{\sqrt{2}}\left[\log\left|\operatorname{cosec}\left(\alpha+\frac{\pi}{2}\right)-\cot\left(\frac{\pi}{2}+\alpha\right)\right|\right] - \log\left|\operatorname{cosec}(0+\alpha)-\cot(0+\alpha)\right|$$

$$2I = \frac{1}{\sqrt{2}}\left[\log|\sec\alpha+\tan\alpha| - \log|\operatorname{cosec}\alpha-\cot\alpha|\right]$$

$$2I = \frac{1}{\sqrt{2}}\left[\log\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{4}-\cot\frac{\pi}{4}\right|\right]$$

$$2I = \frac{1}{\sqrt{2}}\left[\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}+1\right|\right]$$

$$2I = \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$$

$$2I = \frac{1}{\sqrt{2}}\log\left|\frac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}\right|$$

$$2I = \frac{1}{\sqrt{2}}\log\left|\frac{(\sqrt{2}+1)^2}{1}\right|$$

$$2I = \frac{1}{\sqrt{2}}2\log\left|\sqrt{2}+1\right|$$

$\therefore$ $I = \frac{1}{\sqrt{2}}\log\left|\sqrt{2}+1\right|$

5) $\int_0^3 \frac{dx}{x+\sqrt{9-x^2}}$

Ans. $I = \int_0^3 \frac{dx}{x+\sqrt{9-x^2}}$

$x = 3\sin\theta$
$dx = 3\cos\theta\, d\theta$
$x = 3\sin\theta$
When $x = 3$; $x = 0$
$3 = 3\sin\theta$; $0 = 3\sin\theta$
$1 = \sin\theta$; $0 = \sin\theta$

$\theta = \frac{\pi}{2}$ $\therefore$ $\theta = 0$

$$I = \int_0^{\pi/2} \frac{3\cos\theta\, d\theta}{3\sin\theta + \sqrt{9-9\sin^2\theta}}$$

$$I = \int_0^{\pi/2} \frac{3\cos\theta\, d\theta}{3\sin\theta + 3\cos\theta}$$

$$I = \int_0^{\pi/2} \frac{3\cos\theta\, d\theta}{3(\sin\theta + \cos\theta)}$$

$$I = \int_0^{\pi/2} \frac{\cos\theta\, d\theta}{\sin\theta + \cos\theta} \quad ....(i)$$

Replace θ by $\frac{\pi}{2}-\theta$

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-\theta\right)d\theta}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)}$$

$$I = \int_0^{\pi/2} \frac{\sin\theta\, d\theta}{\cos\theta + \sin\theta} \quad ....(ii)$$

Adding (i) and (ii)

$\therefore$ $$2I = \int_0^{\pi/2} \frac{(\cos\theta + \sin\theta)\, d\theta}{(\cos\theta + \sin\theta)}$$

$$2I = [\theta]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$2I = \frac{\pi}{2}$$

$\therefore$ $I = \frac{\pi}{4}$

6) $\int_0^{\pi/2} \sin 2x \log(\tan x)\, dx$

Ans. $I = \int_0^{\pi/2} \sin 2x \log(\tan x)\, dx \quad ....(i)$

Replace x by $\frac{\pi}{2} - x$

$$I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2}-x\right)\log\left(\tan\left(\frac{\pi}{2}-x\right)\right) dx$$

$$I = \int_0^{\pi/2} \sin(\pi - 2x)\log(\cot x)\, dx$$

$$I = \int_0^{\pi/2} \sin 2x\, \log(\cot x)\, dx \quad ....(ii)$$

Adding (i) and (ii)

$\therefore$ $2I = \int_0^{\pi/2} \sin 2x\left[\log(\tan x) + \log(\cot x)\right]dx$

$\therefore$ $2I = \int_0^{\pi/2} \sin 2x \log(\tan x \cot x)\, dx$

$\therefore$ $2I = \int_0^{\pi/2} \sin 2x \log(1)\, dx$

$\therefore$ $2I = 0$ $\quad [\because \log 1 = 0]$

$\therefore$ $I = 0$

BOARD PROBLEMS

1) $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\, dx$

Ans. $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\, dx \quad .....(i)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}}\,dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-a+x}}\,dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}\,dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} \qquad \text{.....(ii)}$$

Adding (i) and (ii), we have

$$2I = \int_0^a \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}+\frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}\right)dx$$

$\therefore$ $$2I = \int_0^a dx$$

$\therefore$ $$I = (x)_0^a$$

$\therefore$ $$I = \frac{1}{2}(a)$$

$\therefore$ $$I = \frac{a}{2}$$

2) $$\int_0^1 x^2(1-x)^{3/2}\,dx$$

Ans. $$I = \int_0^1 x^2(1-x)^{3/2}\,dx$$

$$I = \int_0^1 (1-x)^2\left[1-(1-x)\right]^{3/2}\,dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^1 \left(1-2x+x^2\right)(1-1+x)^{3/2}\,dx$$

$$I = \int_0^1 \left(1-2x+x^2\right)x^{3/2}\,dx$$

$$I = \int_0^1 \left(x^{3/2}-2x^{5/2}+x^{7/2}\right)dx$$

$$I = \int_0^1 x^{3/2}\,dx - 2\int_0^1 x^{5/2}\,dx + \int_0^1 x^{7/2}\,dx$$

$$I = \frac{2}{5}\left(x^{5/2}\right)_0^1 - 2\times\frac{2}{7}\left(x^{7/2}\right)_0^1 + \frac{2}{9}\left(x^{9/2}\right)_0^1$$

$$I = \frac{2}{5}-\frac{4}{7}+\frac{2}{9}$$

$$I = \frac{126-180+70}{315}$$

$$I = \frac{196-180}{315}$$

$$I = \frac{16}{315}$$

3) $$\int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$$

Let $x = \sin\theta$

$\therefore$ $dx = \cos\theta\; d\theta$

When $x = 1\ \theta = \frac{\pi}{2}$

$x = 0\ \ \theta = 0$

$$I = \int_0^{\pi/2} \frac{\cos\theta\,d\theta}{\sin\theta+\cos\theta} \qquad \text{....(i)}$$

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-\theta\right)d\theta}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)}$$

$$\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\pi/2} \frac{\sin\theta\,d\theta}{\cos\theta+\sin\theta} \qquad \text{....(ii)}$$

Adding (i) and (ii)

$$2I = \int_0^{\pi/2} \left(\frac{\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta}{\cos\theta+\sin\theta}\right)d\theta$$

$\therefore$ $$2I = \int_0^{\pi/2} \left(\frac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta}\right)d\theta$$

$\therefore \quad 2I = \int_0^{\pi/2} d\theta$

$\therefore \quad 2I = (\theta)_0^{\pi/2}$

$\therefore \quad 2I = \frac{\pi}{2}$

$\therefore \quad I = \frac{\pi}{4}$

4) $\int_0^{\pi/2} \cos^2 x \, dx$

Ans. $I = \int_0^{\pi/2} \cos^2 x \, dx$ ....(i)

$I = \int_0^{\pi/2} \cos^2\left(\frac{\pi}{2} - x\right) dx$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$I = \int_0^{\pi/2} \sin^2 x \, dx$ ....(ii)

Adding (i) and (ii), we have

$2I = \int_0^{\pi/2} \left(\sin^2 x + \cos^2 x\right) dx$

$\therefore \quad 2I = \int_0^{\pi/2} dx$

$\therefore \quad 2I = (x)_0^{\pi/2}$

$\therefore \quad 2I = \frac{\pi}{2}$

$\therefore \quad I = \frac{\pi}{4}$

5) $\int_0^{\pi/2} \sin^2 x \, dx$

Ans. $I = \int_0^{\pi/2} \sin^2 x \, dx$ ....(i)

$I = \int_0^{\pi/2} \sin^2\left(\frac{\pi}{2} - x\right) dx$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$I = \int_0^{\pi/2} \cos^2 x \, dx$ ...(ii)

Adding (i) and (ii) we have

$2I = \int_0^{\pi/2} \left(\sin^2 x + \cos^2 x\right) dx$

$2I = \int_0^{\pi/2} dx$

$2I = (x)_0^{\pi/2}$

$2I = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

6) $\int_0^{\pi/2} \frac{\cos x \, dx}{\sin x + \cos x}$ ...(i)

Ans. $I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$

$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

$I = \int_0^{\pi/2} \frac{\sin x \, dx}{\cos x + \sin x}$ ...(ii)

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2} \left(\frac{\sin x}{\cos x + \sin x} + \frac{\cos x}{\sin x + \cos x} \right) dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = (x)_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

7) $\displaystyle\int_0^{\pi/2} \frac{dx}{1 + \tan x}$

Ans. $\displaystyle I = \int_0^{\pi/2} \frac{dx}{1 + \tan x}$

$$I = \int_0^{\pi/2} \frac{dx}{1 + \frac{\sin x}{\cos x}}$$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \quad \text{....(i)}$$

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx \right]$$

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$$

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2} \left(\frac{\sin x}{\cos x + \sin x} + \frac{\cos x}{\sin x + \cos x} \right) dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = (x)_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

8) $\displaystyle\int_0^{\pi/2} \frac{dx}{1 + \cot x}$

Ans. $\displaystyle I = \int_0^{\pi/2} \frac{dx}{1 + \cot x}$

$$I = \int_0^{\pi/2} \frac{dx}{1 + \frac{\cos x}{\sin x}}$$

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad \text{...(i)}$$

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx \right]$$

$$I = \int_0^{\pi/2} \frac{\cos x\,dx}{\cos x + \sin x} \quad \text{...(ii)}$$

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2} \left(\frac{\sin x}{\cos x + \sin x} + \frac{\cos x}{\sin x + \cos x} \right) dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = (x)_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

9) $\int_0^{\pi/2} \frac{\sqrt{\sin x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}}$

Ans. $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}}$ ...(i)

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(ii)

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2}\left(\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right)dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = (x)_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

10) $\int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}dx$

Ans. $I = \int_0^{\pi/2} \frac{\sqrt{\sec x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}dx$ ...(i)

$$I = \int_0^{\pi/2} \frac{\sqrt{\sec\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sec\left(\frac{\pi}{2}-x\right)}+\sqrt{\operatorname{cosec}\left(\frac{\pi}{2}-x\right)}}dx$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$I = \int_0^{\pi/2} \frac{\sqrt{\operatorname{cosec} x}}{\sqrt{\sec x}+\sqrt{\sec x}}dx$ ...(ii)

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2}\left(\frac{\sqrt{\sec x}}{\sqrt{\sec x}+\sqrt{\operatorname{cosec} x}}+\frac{\sqrt{\operatorname{cosec} x}}{\sqrt{\operatorname{cosec} x}+\sqrt{\sec x}}\right)dx$$

$$2I = \int_0^{\pi/2} dx$$

$$2I = (x)_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

11) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$

Ans. $I = \int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$

Let $x = a\sin\theta$ $\therefore$ $dx = a\cos\theta\,d\theta$

when $x = a$, $\theta = \frac{\pi}{2}$

$x = 0$, $\theta = 0$

$\therefore$ $I = \int_0^{\pi/2} \frac{a\cos\theta\,d\theta}{a\sin\theta + a\cos\theta}$

$I = \int_0^{\pi/2} \frac{\cos\theta\,d\theta}{\sin\theta+\cos\theta}$ ...(i)

$$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-\theta\right)d\theta}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)}$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$\therefore$ $I = \int_0^{\pi/2} \frac{\sin\theta\,d\theta}{\cos\theta+\sin\theta}$ ...(ii)

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2} \left(\frac{\cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta}{\cos\theta + \sin\theta} \right) d\theta$$

$$\therefore \quad 2I = \int_0^{\pi/2} d\theta$$

$$\therefore \quad 2I = (x)_0^{\pi/2}$$

$$\therefore \quad 2I = \frac{\pi}{2}$$

$$\therefore \quad I = \frac{\pi}{4}$$

12) $\displaystyle\int_0^{\pi/2} \frac{\sin^2 x\,dx}{(\sin x + \cos x)^2}$

Ans. $$I = \int_0^{\pi/2} \frac{\sin^2 x\,dx}{(\sin x + \cos x)^2} \quad \ldots(i)$$

$$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\left[\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)\right]^2}$$

$$\left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{(\cos x + \sin x)^2} dx \quad \ldots(ii)$$

Adding (i) and (ii), we have

$$2I = \int_0^{\pi/2} \left[\frac{\sin^2 x}{(\sin x + \cos x)^2} + \frac{\cos^2 x}{(\cos x + \sin x)^2}\right] dx$$

$$\therefore \quad 2I = \int_0^{\pi/2} \frac{dx}{(\sin x + \cos x)^2} dx$$

Now

$$\cos x + \sin x = \sqrt{2}\left(\cos x.\frac{1}{\sqrt{2}} + \sin x.\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\left(\cos x \cos\frac{\pi}{4} + \sin 4x \sin\frac{\pi}{4}\right)$$

$$= \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$$

$$\therefore \quad 2I = \int_0^{\pi/2} \frac{dx}{\left\{\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)\right\}^2}$$

$$\therefore \quad 2I = \frac{1}{2}\int_0^{\pi/2} \sec^2\left(x - \frac{\pi}{4}\right) dx$$

$$\therefore \quad 2I = \frac{1}{2}\left\{\tan\left(x - \frac{\pi}{4}\right)\right\}_0^{\pi/2}$$

$$\therefore \quad 2I = \frac{1}{4}\left[\tan\left(\frac{\pi}{2} - \frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{4}\right)\right]$$

$$\therefore \quad I = \frac{1}{4}\left[\tan\frac{\pi}{4} + \tan\frac{\pi}{4}\right]$$

$$\therefore \quad I = \frac{1}{4} \times 2$$

$$\therefore \quad I = \frac{1}{2}$$

13) $\displaystyle\int_0^{\pi/4} \log(1 + \tan x)\,dx$

Ans. $$I = \int_0^{\pi/4} \log(1 + \tan x)\,dx$$

$$I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$$

$$I = \int_0^{\pi/4} \log\left[1 + \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right] dx$$

$$I = \int_0^{\pi/4} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right] dx.$$

$$I = \int_0^{\pi/4} \log\left[\frac{1 + \tan x + 1 - \tan x}{1 + \tan x}\right] dx$$

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$$

$$\therefore \quad I = \int_0^{\pi/4} \left[\log 2 - \log(1+\tan x)\right]dx$$

$$\therefore \quad I = \log 2 \int_0^{\pi/4} dx \int_0^{\pi/4} \log(1+\tan x)\,dx$$

$$\therefore \quad I = \log 2 (x)_0^{\pi/4} \quad \ldots(i)$$

$$\therefore \quad 2I = \frac{\pi}{4}\log 2$$

$$\therefore \quad I = \frac{\pi}{8}\log 2$$

14) $\displaystyle\int_0^{\pi/2} \frac{\sin^2 x\,dx}{\sin x + \cos x}$

Ans. $\displaystyle I = \int_0^{\pi/2} \frac{\sin^2 x\,dx}{\sin x + \cos x} \quad \ldots(i)$

$$\therefore \quad I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx$$

$$\left[\because \int_0^a f(a)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}dx \quad \ldots(ii)$$

Adding (i) and (ii) we have,

$$2I = \int_0^{\pi/2} \left(\frac{\sin^2 x}{\sin x + \cos x} + \frac{\cos^2 x}{\sin x + \cos x}\right)dx$$

$$2I = \int_0^{\pi/2} \frac{dx}{\sin x + \cos x}$$

Now

$$\sin x + \cos x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$= \frac{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}{\sec^2\frac{x}{2}}$$

$$\therefore \quad 2I = \int_0^{\pi/2} \frac{dx}{\dfrac{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}{\sec^2\frac{x}{2}}}$$

$$\therefore \quad 2I = \int_0^{\pi/2} \frac{\sec^2\frac{x}{2}\,dx}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

Let $\tan\frac{\pi}{2} = z \quad \sec^2\frac{\pi}{2}dx = 2dz$

When $x = \frac{\pi}{2}$, $z = 1$

$x = 0, \quad z = 0$

$$2I = \int_0^1 \frac{2\,dz}{2z+1-z^2}$$

$$= 2\int_0^1 \frac{dz}{2-\left(z^2-2z+1\right)}$$

$$\therefore \quad 2I = 2\int_0^1 \frac{dz}{\left(\sqrt{2}\right)^2 - (z-1)^2}$$

$$\therefore \quad 2I = \frac{1}{2\sqrt{2}}\left[\log\left|\frac{\sqrt{2}+z-1}{\sqrt{2}-z+1}\right|\right]_0^1$$

$$\therefore \quad 2I = \frac{1}{2\sqrt{2}}\left[\log\frac{\sqrt{2}}{\sqrt{2}} - \log\frac{\sqrt{2}-1}{\sqrt{2}-1}\right]$$

$$\therefore \quad I = \frac{1}{2\sqrt{2}}\left[-\log\left\{\frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}\right\}\right]$$

$$\therefore \quad I = \frac{1}{2\sqrt{2}}\left[-\log\frac{\left(\sqrt{2}-1\right)^2}{2-1}\right]$$

$$\therefore \quad I = -\frac{1}{2\sqrt{2}}.2\log\left(\sqrt{2}-1\right)$$

$$\therefore \quad I = -\frac{1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$$

$\therefore \quad I = -\frac{1}{\sqrt{2}}\log 1 - \frac{1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$

$\therefore \quad I = -\frac{1}{\sqrt{2}}\log\left(\frac{1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$

$\therefore \quad I = \frac{1}{\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$

$\therefore \quad I = \frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$

GROUP (E)-HOME WORK PROBLEMS

1) $\int_{-1}^{1}\frac{1-x^2}{1+x^2}\,dx$

Ans. $I = \int_{-1}^{1}\frac{1-x^2}{1+x^2}\,dx$

$f(x) = \frac{1-x^2}{1+x^2}$

$f(-x) = \frac{1-\left(-x^2\right)}{1+\left(-x^2\right)} = f(-x) = \frac{1-x^2}{1+x^2} = f(x)$

$\therefore$ f(x) is even function

$I = 2\int_{0}^{1}\frac{1-x^2}{1+x^2}dx \quad = 2\int_{0}^{1}\frac{\left(-\left(x^2+1\right)+2\right)}{1+x^2}dx$

$I = 2\left[\int_{0}^{1}-dx + 2\int_{0}^{1}\frac{dx}{1+x^2}\right]$

$I = 2\left[-x+2\tan^{-1}x\right]_0^1$

$I = 2\left[\left[-1+2\tan^{-1}1\right]-0\right]$

$I = 2\left[-1+2\times\frac{\pi}{4}\right]$

$I = \pi - 2$

2) $\int_{-1}^{1}\frac{x^2}{x^2+1}\,dx$

Ans. $f(x) = \frac{x^2}{x^2+1}$

$f(-x) = \frac{(-x)^2}{(-x)^2+1} = \frac{x^2}{x^2+1}$

$f(-x) = f(x)$

$\therefore$ f (x) is even function

$\therefore \quad I = 2\int_{0}^{1}\frac{x^2}{x^2+1}\,dx$

$\therefore \quad I = 2\int_{0}^{1}\frac{x^2+1-1}{x^2+1}\,dx$

$I = 2\int_{0}^{1}\left(1-\frac{1}{x^2+1}\right)dx$

$I = 2\left[x-\tan^{-1}x\right]_0^1$

$I = 2\left[1-\tan^{-1}1\right]$

$I = 2\left[1-\frac{\pi}{4}\right]$

$\therefore \quad I = 2-\frac{\pi}{2}$

3) $\int_{-a}^{a}\frac{x^3}{4-x^2}\,dx$

Ans. $I = \int_{-a}^{a}\frac{x^3}{4-x^2}\,dx$

$f(x) = \frac{x^3}{4-x^2}$

$f(-x) = \frac{(-x)^3}{4-(-x)^2} = \frac{-x^3}{4-x^2}$

$f(-x) = -f(x)$

$\therefore$ f (x) is odd function

$I = \int_{-a}^{a}\frac{x^3}{4-x^2}dx$

$I = 0$

4) $\int_{-\pi/2}^{\pi/2}\cos^6 x\,dx$

Ans. $I = \int_{-\pi/2}^{\pi/2}\cos^6 x\,dx$

$f(x) = \cos^6 x$

$f(-x) = \cos^6(-x) = \cos^6 x$

$\therefore$ f (x) is even function

$$I = 2\int_0^{\pi/2} \cos^6 x\ dx$$

$$\left[\int \cos^n x\ dx = \frac{\cos^{n-1} x \sin x}{4} + \frac{n-1}{n} \int \cos^{n-2} x\,dx\right]$$

$$I = 2\left[\frac{\cos^5 x \sin x}{6} + \frac{5}{6}\left[\frac{\cos^3 x \sin x}{4} + \frac{3}{4}\int \cos^2 x\,dx\right]\right]$$

$$I = 2\left[\frac{\cos^5 x \sin x}{6} + \frac{5}{24}\cos^3 x \sin x + \frac{15}{24}\int \cos^2 x\,dx\right]$$

$$I = 2\left[\frac{\cos^5 x \sin x}{6} + \frac{5}{24}\cos^3 x \sin x + \frac{15}{24}\left(\frac{\cos^1 x \sin x}{2} + \frac{1}{2}\int \cos^0 x\right)\right]$$

$$I = 2\left[\frac{\cos^5 x \sin x}{6} + \frac{5}{24}\cos^3 x \sin x + \frac{15}{48} \cos x \sin x + \frac{15}{48} x\right]_0^{\pi/2}$$

$$I = 2\left[\left[0 + 0 + 0 + \frac{15}{48}\frac{\pi}{2}\right] - 0\right]$$

$$I = \frac{2 \times 15\pi}{48 \times 2}$$

$$I = \frac{5\pi}{16}$$

5) $\int_{-\pi/4}^{\pi/4} \tan^3 x \sec x\ dx$

$f(x) = \tan^3 x \sec x$

$f(-x) = \tan^3(-x) \sec(-x)$

$= -\tan^3 x \sec x$

$f(-x) = -f(x)$

$\therefore$ f (x) is odd function

$\therefore \quad I = \int_{-\pi/4}^{\pi/4} \tan^3 x \sec x\ dx = 0$

GROUP (F)-HOME WORK PROBLEMS

1) $\int_0^{\pi/2} \log(\cos x)\,dx$

Ans. $I = \int_0^{\pi/2} \log(\cos x)\,dx \qquad \ldots(i)$

Replace x by $\frac{\pi}{2} - x$

$$I = \int_0^{\pi/2} \log\left(\cos\left(\frac{\pi}{2} - x\right)\right)dx$$

$$I = \int_0^{\pi/2} \log(\sin x)\,dx \qquad \ldots(ii)$$

Adding (i) and (ii)

$$\therefore \quad 2I = \int_0^{\pi/2} \log(\sin x) + \log(\cos x)\,dx$$

$$\therefore \quad 2I = \int_0^{\pi/2} \log\left(\frac{2\sin x \cos x}{2}\right)dx$$

$$2I = \int_0^{\pi/2} \log(\log \sin 2x - \log 2)\,dx$$

$$2I = \int_0^{\pi/2} \log \sin 2x\,dx - \log 2 \int_0^{\pi/2} dx$$

$$2I = I_1 - I_2 \qquad \ldots(i)$$

$$\therefore \quad I_1 = \frac{1}{2}\int_0^{\pi} \log \sin \pi\ d\pi$$

$$= \frac{1}{2}\int_0^{2.\frac{\pi}{2}} \log \sin \pi\ d\pi$$

$$\left[\text{If } f(2a - x) = f(x) \text{ i.e. } \sin\left(2.\frac{\pi}{2} - \pi\right) = \sin \pi\right]$$

$$= \frac{1}{2}.2\int_0^{\pi/2} \log \sin \pi\ d\pi$$

$$= \int_0^{\pi/2} \log \sin \pi\ d\pi \left[\int_a^b f(x)\,dx = \int_a^b f(y)\,dy\right]$$

$$= \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx \quad \left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,da\right]$$

$$= \int_0^{\pi/2} \log \cos x\,dx$$

$$= I \qquad \text{....(ii)}$$

$$I_2 = \log 2 \int_0^{\pi/2} dx$$

$$= (x)_0^{\pi/2} \log 2 \qquad \text{...(iii)}$$

$$= \frac{\pi}{2}\log 2$$

from (i)

$2I = \log 2 \ \frac{\pi}{2}\log 2$ (by ii and iii)

$\therefore \quad I = -\frac{\pi}{2}\log 2$

$\therefore \quad I = -\frac{\pi}{2}\log 2$

$\therefore \quad I = \frac{\pi}{2}\log 1 - \frac{\pi}{2}\log 2$

$$= \frac{\pi}{2}\log\left(\frac{1}{2}\right)$$

2) $\int_0^1 \sin x \cos^4 x\,dx$

Ans. $I = \int_0^1 \sin x \cos^4 x\,dx$

$\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

$\cos x = t$

When $x = 1$; $x = 0$

$\cos 1 = t$; $\cos 0 = t$

$0 = t$; $\frac{\pi}{2} = t$

$$I = \int_{\frac{\pi}{2}}^{0} t^4 (-dt)$$

$$I = \int_0^{\pi/2} t^4 dt$$

$$I = \left[\frac{t^5}{5}\right]_0^{\pi/2}$$

$$I = \frac{1}{5}\left[\left[\frac{\pi}{2}\right]^5 - 0\right]$$

$$I = \frac{1}{5} \times \frac{\pi^5}{32}$$

$$I = \frac{\pi^5}{160}$$

3) $I = \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$

Ans. $I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1 + \sin x(\pi - x)} dx$

$\therefore \quad I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1 + \sin x} dx$

$\therefore \quad I = \pi\int_0^{\pi} \frac{\sin x}{1 + \sin x} dx - \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$

$\therefore \quad 2I = \pi\int_0^{\pi} \frac{\sin x\,dx}{1 + \sin x} = \pi\int_0^{\pi} \frac{\sin x(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$

$\therefore \quad 2I = \pi\int_0^{\pi} \frac{\sin x(1 - \sin x)}{\cos^2 x} dx$

$\therefore \quad 2I = \pi\int_0^{\pi} \left(\frac{\sin x - \sin^2 x}{\cos^2 x}\right) dx$

$\therefore \quad 2I = \pi\int_0^{\pi} \left(\sec x \tan x - \tan^2 x\right) dx$

$\therefore \quad 2I = \pi\left[\int_0^{\pi} \sec x \tan x\,dx - \int_0^{\pi} \tan^2 x\,dx\right]$

$\therefore \quad 2I = \pi\left[\int_0^{\pi} \sec x \tan x\,dx - \int_0^{\pi} \sec^2 x\,dx + \int_0^{\pi} dx\right]$

$\therefore \quad 2I = \pi\left[(\sec x)_0^{\pi} - (\tan x)_0^{\pi} + (x)_0^{\pi}\right]$

$\therefore \quad 2I = \pi\left[(-1-1) - (0-0) + \pi\right]$

$\therefore \quad 2I = \frac{\pi}{2}(-2+\pi) = \frac{\pi}{2}(\pi-2)$

4) $\int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$

Ans. $I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$

$\therefore \quad I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-x\right)dx}{\cos x+\sin x}$

$\therefore \quad I = \frac{\pi}{2}\int_0^{\pi/2} \frac{dx}{\sin x+\cos x} - \int_0^{\pi/2} \frac{x\,dx}{\sin x+\cos x}$

$\therefore \quad 2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{dx}{\sin x+\cos x}$

$\sin x + \cos x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$= \frac{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\therefore \quad 2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$

$\therefore \quad 2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx$

Let $\tan\frac{\alpha}{2} = \pi$

$\left(\sec^2\frac{x}{2}\right).\frac{1}{2}dx = d\pi$

$\therefore \quad \sec^2.\frac{x}{2}dx = 2d\pi$

when $x = \frac{\pi}{2}, \pi = 1$

$x = 0 \quad \pi = 0$

$\therefore \quad 2I = \frac{\pi}{2}\int_0^1 \frac{2\,d\pi}{2\pi+1-\pi^2}$

$\therefore \quad 2I = \frac{\pi}{2}.2\int_0^1 \frac{d\pi}{2-\left(\pi^2-2\pi+1\right)}$

$\therefore \quad 2I = \pi\int_0^1 \frac{d\pi}{\left(\sqrt{2}\right)^2-(\pi-1)^2}$

$\therefore \quad 2I = \pi.\frac{1}{2\sqrt{2}}\left[\log\left|\frac{\sqrt{2}+\pi-1}{\sqrt{2}-\pi+1}\right|\right]_0^1$

$\therefore \quad 2I = \frac{\pi}{2\sqrt{2}}\left(\log\frac{\sqrt{2}}{\sqrt{2}} - \log\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$

$\therefore \quad 2I = \frac{\pi}{2\sqrt{2}}\left[0-\log\frac{\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}\right]$

$\therefore \quad 2I = \frac{\pi}{2\sqrt{2}}\log\frac{\left(\sqrt{2}-1\right)^2}{2-1}$

$\therefore \quad 2I = \frac{\pi}{2\sqrt{2}}.2\log\sqrt{2}-1$

$\therefore \quad I = -\frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}-1\right)$

$I = \frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$

5) $\int_0^{\pi/2} \log(\sin x)\,dx$

Ans. $I = -\int_0^{\pi/2} \log(\sin x)\,dx$ ...(i)

$= -\int_0^{\pi/2} \log\sin\left(\frac{\pi}{2}-x\right)dx$

$= -\int_0^{\pi/2} \log\cos x\,dx$ ...(ii)

Adding (i) and (ii) we have

$2I = -\int_0^{\pi/2} (\log\sin x+\log\cos x)\,dx$

$\therefore \quad 2I = -\int_0^{\pi/2} \log\left(\frac{2\sin x\cos x}{2}\right) dx$

$\therefore \quad 2I = -\int_0^{\pi/2} (\log\sin 2x - \log 2)\, dx$

$\therefore \quad 2I = -\int_0^{\pi/2} \log\sin 2x\, dx + \log 2\int_0^{\pi/2} dx$

$\therefore \quad 2I = I_1 + I_2$

$I_1 = -\int_0^{\pi/2} \log\sin 2x\, dx$

put $2x = \pi$

$dx = \frac{d\pi}{2}$

when $x = \frac{\pi}{2}$, $z = \pi$

$x = 0, \quad z = 0$

$\therefore \quad I_1 = -\frac{1}{2}\int_0^{\pi} \log\sin z\, dz$

$\therefore \quad I_1 = -\frac{1}{2}\int_0^{\pi} \log\sin x\, dx$

$\therefore \quad I_1 = -\frac{1}{2}\int_0^{2.\frac{\pi}{2}} \log\sin x\, dx$

$\therefore \quad I_1 = -\frac{1}{2}\int_0^{2.\frac{\pi}{2}} \log\sin x\, dx$

$\therefore \quad I_1 \; -\frac{1}{2}.2\int_0^{\pi} \log\sin x\, dx \left[\because f(2a-x) = f(x)\right]$

$\therefore \quad I_1 = I$ ...(iii)

$I_2 = \log 2 \int_0^{\pi/2} dx$

$\therefore \quad I_2 = (x)_0^{\pi/2} \log 2$

$\therefore \quad I_2 = \frac{\pi}{2}\log 2$ .....(iv)

$\therefore \quad 2I = I + \frac{\pi}{2}\log 2$ (by iii and iv)

$\therefore \quad I = \frac{\pi}{2}\log 2$

6) $I = \int_0^{\pi/2} \frac{x\frac{1}{\cos x}dx}{1+\frac{\sin x}{\cos x}}$

Ans. $I = \int_0^{\pi/2} \frac{x\frac{1}{\cos x}dx}{1+\frac{\sin x}{\cos x}}$

$I = \int_0^{\pi/2} \frac{\frac{x}{\cos x}dx}{\frac{\cos x+\sin x}{\cos x}}$

$\therefore \quad I = \int_0^{\pi/2} \frac{x\,dx}{\sin x+\cos x}$

Refer as Group F class work (5)

5) $\int_0^{\pi/2} \log(\operatorname{cosec} x)\, dx$

$I = \int_0^{\pi/2} -\log(\sin x)\, dx$

$I = -\int_0^{\pi/2} \log(\sin x)\, dx$ ...(i)

Refer as Group F class work (2)

GROUP (G)-HOME WORK PROBLEMS

Q -1) $\int_3^9 \frac{\sqrt[3]{12-x}}{\sqrt[3]{x}+\sqrt[3]{12-x}} dx$

Ans. Let $I = \int_3^9 \frac{\sqrt[3]{12-x}}{\sqrt[3]{x}+\sqrt[3]{12-x}} dx$ ...(i)

using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$= \int_3^9 \frac{\sqrt[3]{12-(12-x)}}{\sqrt[3]{12-x}+\sqrt[3]{12-(12-x)}} dx$

$= \int_3^9 \frac{\sqrt[3]{x}}{\sqrt[3]{12-x}+\sqrt[3]{x}} dx$ ... (ii)

Adding (i) and (ii)

$2I = \int_3^9 \frac{\sqrt[3]{x}+\sqrt[3]{12-x}}{\sqrt[3]{x}++\sqrt[3]{12-x}} dx$

$\therefore \quad 2I = \int_3^9 1\, dx = [x]_3^9$

$\therefore \quad 2I = 6$

$\therefore \quad I = 3$

2) $\int_2^6 \frac{(8-x)^2}{x^2+(8-x)^2}dx$

Ans. Let $I = \int_2^6 \frac{(8-x)^2}{x^2+(8-x)^2}dx$ ...(i)

$$= \int_2^6 \frac{[8-(8-x)]^2}{(8-x)^2+[8-(8-x)]^2}dx$$

$\therefore$ $I = \int_2^6 \frac{x^2}{(8-x)^2+x^2}dx$ ...(ii)

Adding (i) and (ii)

$\therefore$ $2I = \int_2^6 \frac{(8-x)^2+x^2}{x^2+(8-x)^2}dx$

$\therefore$ $2I = \int_2^6 1\,dx = [x]_2^6$

$\therefore$ 2I = 4

$\therefore$ I = 2

3) $2I = \int_1^3 \frac{\sqrt[1]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$

Ans. I = $2I = \int_1^3 \frac{\sqrt[1]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$ ...(i)

using $\int_a^b f(x)dx = \int_a^b (a+b-x)dx$

$I = \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{5+x}}dx$ ...(ii)

Adding (i) and (ii)

$\therefore$ $2I = \int_1^3 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+5}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}dx$

$\therefore$ $2I = \int_1^3 dx = [x]_1^3$

$\therefore$ 2I = 2

I = 1

4) $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx$

Ans. Let $I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx$

$$= \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\sin x/\cos x}}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \text{... (i)}$$

$\int_a^b f(x)dx = \int_a^b (a+b-x)dx$

$\therefore$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$

$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ... (ii)

Adding (i) and (ii)

$\therefore$ $2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

$\therefore$ $2I = \int_{\pi/6}^{\pi/3} dx = [x]_{\pi/6}^{\pi/3}$

$\therefore$ $2I = \frac{\pi}{6}$

$I = \pi\ /\ 12$

5) $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\cot^{3/2} x}$

Ans. $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\cot^{3/2} x}$ ...(i)

$$= \int_{\pi/6}^{\pi/3} \frac{dx}{1+(\cos x/\sin x)^{3/2}}dx$$

$\therefore$ $I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x}dx$ ...(ii)

Adding (i) and (ii)

$\therefore$ $2I = \int_{\pi/6}^{\pi/3} 1\,dx = [x]_{\pi/6}^{\pi/3} = \pi/6$

$\therefore$ $I = \frac{\pi}{12}$

6) $\int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$

Ans. $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$ ...(i)

using $\int_a^b f(x)dx = \int_a^b (a+b-x)dx$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$\therefore$ $I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$ ... (ii)

Adding (i) and (ii)

$\therefore$ $2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$

$\therefore$ $2I = \int_{\pi/6}^{\pi/3} 1\,dx = [x]_{\pi/6}^{\pi/3}$

$\therefore$ $2I = \pi/6$

$\therefore$ $I = \frac{\pi}{12}$

7) $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\ dx$

Ans. $I = \int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\ dx$ ... (i)

$= \int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{9-(9-x)}}\ dx$

$= \int_2^7 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}\ dx$ ... (ii)

Adding (i) and (ii)

$\therefore$ $2I = \int_2^7 \frac{\sqrt{9-x}+\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\ dx$

$\therefore$ $2I = \int_2^7 1.\ dx = [x]_2^7$

$\therefore$ $2I = 5$

$\therefore$ $I = 5/2$

8) $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\ dx$

Ans. $I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\ dx$ ... (i)

$I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-(7-x)}+\sqrt{7-x}}\ dx$

$\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\ dx$ ... (ii)

Adding (i) and (ii)

$\therefore$ $2I = \int_2^5 \frac{\sqrt{7-x}+\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\ dx$

$\therefore$ $2I = \int_2^5 1\ dx = [x]_2^5$

$\therefore$ $2I = 3$

$\therefore$ $I = 3/2$

GROUP (H)-HOME WORK PROBLEMS

1) $\int_2^5 (4x+3)\ dx$

Ans. Let $I = \int_2^5 (4x+3)\ dx$

Let $f(x) = 4x + 3$; $a = 2$, $b = 5$

$h = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$

$\therefore$ $nh = 3$

$\therefore$ $\int_2^5 f(x)\,dx = \lim_{n\to\infty} \sum_{r=1}^{n} h\ f(a+rh)$

$= \lim_{n\to\infty} \sum_{r=1}^{n} h\ f(2+rh)$

$= \lim_{n\to\infty} \sum_{r=1}^{n} h\left(4(2+rh)+3\right)$

$= \lim_{n\to\infty} \sum_{r=1}^{n} h(11+4rh)$

$= \lim_{n\to\infty}\left[\sum_{r=1}^{n} 11h + 4h^2\sum_{r=1}^{n} r\right]$

$= \lim_{n\to\infty}\left[11nh + 4h^2 \times \frac{n(n+1)}{2}\right]$

$$= \lim_{n\to\infty}\left[11(3)+4h^2\times n^2\,\frac{\left(1+\frac{1}{n}\right)}{2}\right]$$

$$=\left[33+4(3)^2\times\frac{(1+0)}{2}\right]$$

$$=33+2(9)$$

= 33 + 18

= 51

2) $\int_0^3 (2x+3)\,dx$

Ans. Let, I = $\int_0^3 (2x+3)\,dx$

Let f(x) = 2x + 3, a = 0, b = 3

$h=\frac{b-a}{n}=\frac{3-0}{n}$

∴ nh = 3

∴ $\int_0^3 f(x)\,dx = \lim_{n\to\infty}\sum_{r=1}^{n} h\,f(a+rh)$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} h\,f(rh) \qquad [\because a=0]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} h\left(2(rh)+3\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(2rh^2+3h\right)$$

$$= \lim_{n\to\infty}\left[2h^2\sum_{r=1}^{n} r+3h\sum_{r=1}^{n}1\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left[2h^2\times\frac{n(n+1)}{2}+3h\times n\right]$$

$$= \lim_{n\to\infty}\left[\frac{2h^2\times n^2\left(1+\frac{1}{n}\right)}{2}+3nh\right]$$

$$=\frac{2\times(3)^2(1+0)^2}{2}+3(3)$$

= 9 + 9

= 18

3) $\int_0^1 (2x+3)\,dx$

Ans. Let, I = $\int_0^1 (2x+3)\,dx$

f(x) = 2x + 3, a = 0, b = 1

∴ $\int_0^1 f(x)\,dx = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{r=1}^{n}\left(2\left(\frac{r}{n}\right)+3\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\left[\sum_{r=1}^{n}\frac{2r}{n}+3\right]\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{n^2}\sum_{r=1}^{n} r+\frac{3}{n}\sum_{r=1}^{n}1\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{n^2}\times\frac{n(n+1)}{2}+\frac{3}{n}\times n\right]$$

$$= \lim_{n\to\infty}\left[\frac{2}{n^2}\times\frac{n^2\left(1+\frac{1}{n}\right)}{2}+3\right]$$

$$=\frac{2(1+0)}{2}+3$$

= 1 + 3

= 4

4) $\int_0^2 (3x+5)\,dx$

Ans. Let,I = $\int_0^2 (3x+5)\,dx$

Let f(x) = 3x + 5, a = 0, b = 2

∴ $h=\frac{b-a}{n}=\frac{2-0}{n}=\frac{2}{n}$

∴ nh = 2

∴ $\int_0^2 f(x)\,dx = \lim_{n\to\infty}\sum_{r=1}^{n} h\,f(a+rh)$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} h\,f(rh) \qquad [\because a=0]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} h\left(3(rh)+5\right)$$

$$= \lim_{n\to\infty}\left[3h^2\sum_{r=1}^{n} r+5h\sum_{r=1}^{n}1\right]$$

$$= \lim_{n\to\infty}\left[3h^2\times\frac{n(n+1)}{2}+5hn\right]$$

$$= \lim_{n\to\infty}\left[\frac{3h^2\times n^2\left(1+\frac{1}{n}\right)}{2}+5hn\right]$$

$$= \frac{3(2)^2(1+0)}{2}+5(2)$$

$= 6 + 10$

$= 16$

5) $\int_0^2 \left(2x^2+5\right)dx$

Ans. Let I = $\int_0^2 \left(2x^2+5\right)dx$

Let $f(x) = 2x^2 + 5$, a = 0, b = 2

$\therefore$ $h = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

$\therefore$ n h = 2

$\therefore$ $\int_0^2 f(x)dx = \lim_{n\to\infty}\sum_{r=1}^{n} hf(a+rh)$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} hf(rh) \qquad [\because a=0]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} h\left(2(rh)^2+5\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(2h^3r^2+5h\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left[2h^3\sum_{r=1}^{n} r^2+5h\sum_{r=1}^{n}1\right]$$

$$= \lim_{n\to\infty}\left[2h^3\times\frac{n(n+1)(2n+1)}{6}+5hn\right]$$

$$= \lim_{n\to\infty}\left[\frac{2h^3\times n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}\right]+5(2)$$

$$= \frac{2(2)^3(1+0)(2+0)}{6}+10$$

$$= \frac{32}{6}+10$$

$$= \frac{46}{3}$$

6) $\int_0^2 \left(3x^2+5\right)dx$

Ans. Let, I = $\int_0^2 \left(3x^2+5\right)dx$

let $f(x) = 3x^2 + 5$, a = 0, b = 2

$\therefore$ $h = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

$\therefore$ nh = 2

$\int_0^2 f(x)dx = \lim_{n\to\infty}\sum_{r=1}^{n} hf(a+rh)$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} hf(rh) \qquad [\because a=0]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} h\left(3(rh)^2+5\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(3r^2h^3+5h\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left[3h^3\sum_{r=1}^{n} r^2+5h\sum_{r=1}^{n}1\right]$$

$$= \lim_{n\to\infty}\left[3h^3\times\frac{n(n+1)(2n+1)}{6}+5h\times n\right]$$

$$= \lim_{n\to\infty}\left[\frac{3h^3\times n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}+5nh\right]$$

$$= \frac{3(2)^3(1+0)(2+0)}{6}+5(2)$$

$= 8 + 10$

$= 18$